Extra Difficulty Assignment: “All Languages are Decidable”

We give a (supposed) proof that all languages are decidable. We know that the proof must be incorrect, since there are undecidable languages. Your task is to find a flaw in the proof.

Theorem 1 *Every language is decidable.*

Proof: Let Σ be any alphabet and L any language over Σ. For any integer n, Let Σ^n be the set of all strings of length n over Σ, a finite set of cardinality $|\Sigma|^n$. Let $L_n = L \cap \Sigma^n$, the set of all strings in L of length n. Then L_n is finite, since it cannot be bigger than Σ^n.

Every finite language is decidable. Let P_n be a program which decides L_n. Then the following program decides L.

Read $w \in \Sigma^*$
Let $n = |w|$.
If(P_n accepts w)
    Write 1
Else
    Write 0

Thus, L is decidable. Since L is an arbitrary language, every language is decidable. ∎

Ideas that do not Work

During the exam, many students tried to find the flaw in the proof, but none did. (Although one person had a glimmer of the correct idea.) Here are some of the answers students wrote.

1. “L_n is not finite.” Yes, it is, since it is a subset of the finite set Σ^n.
2. “L_n is not decidable.” Yes, it is. You can find a proof that every finite language is decidable in many places on the internet.
3. “P_n does not exist.” Yes, it does, since by definition, if a language is decidable, it is decided by some program.
4. “The program gets stuck at the if condition.” No it doesn’t, because a program that decides a language halts with any input.

Good Luck!