Computer Science 456/656 Fall 2019

Answers to First Examination September 16, 2019

1. True or False. If the question is currently open, write "O" or "Open."

   (i) **F** Every subset of a regular language is regular.

   (ii) **T** Every DFA is an NFA.

   (iii) **F** Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, for any m and n. Then L is a regular language.

   (iv) **F** Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a regular language.

   (v) **T** The Kleene closure of every regular language is regular.

   (vi) **T** The language consisting of all hexadecimal numerals for positive integers n such that $n \% 13 = 7$ is regular.

   (vii) **T** The complement of every regular language is regular.

   (viii) **T** The union of any two regular languages is regular.

   (ix) **T** There exists a mathematical proposition that is true, but where no proof of the proposition can exist.

   (x) **F** Every language generated by a grammar is regular.

   (xi) **O** There is a $\mathcal{P}$–TIME algorithm which decides whether two regular expressions are equivalent.

   (xii) **T** If x and y are equivalent regular expressions, there is a $\mathcal{P}$–TIME proof that x and y are equivalent.

   (xiii) **F** The set of all decimal numerals for prime numbers is a regular language.

   (xiv) **T** For any non-deterministic finite automaton, there is always a unique minimal deterministic finite automaton equivalent to it.

   (xv) **T** It can always be decided whether two given regular expressions are equivalent.

   (xvi) **T** The complement, over the binary alphabet, of every regular binary language is regular.

   (xvii) **T** If L is regular, then L^R is regular.

   (xviii) **T** Every finite language is regular.

   (xix) **F** The set of all palindromes over the binary alphabet is a regular language.

   (xx) **T** The language of all strings over $\{a, b\}$ which begin and end with the same symbol is regular.

   (xxi) **T** The intersection of any two regular languages is regular.

   (xxii) **F** The set of all strings which could be expressions in a C++ program is a regular langugage.

   (xxiii) **T** There is no computer program that decides whether two given C++ programs are equivalent.

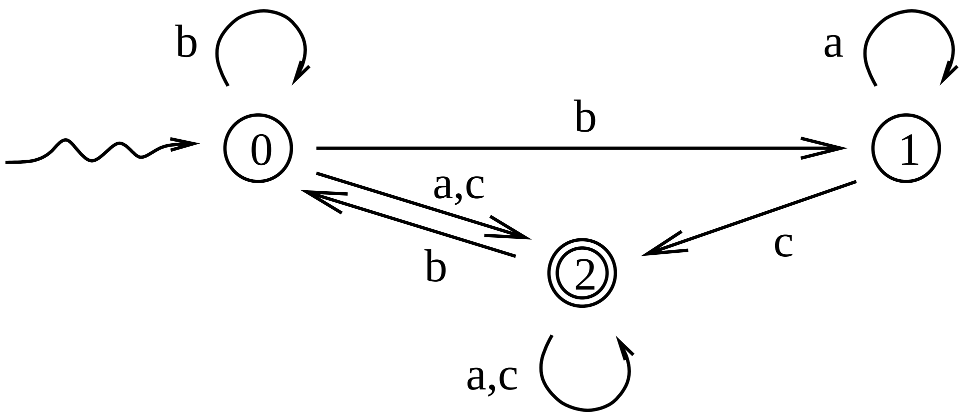


Figure 1: The NFA for Problems 2 and 3.

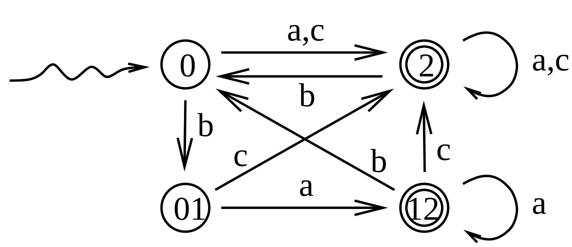
2. [20 points] Give a regular grammar for the language accepted by the NFA shown in Figure 1.

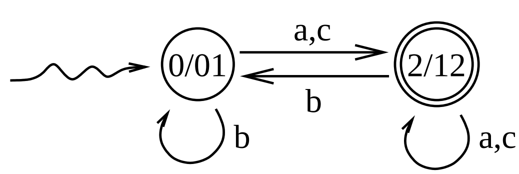
   Let S, A, B be the variables corresponding to the states q_0, q_1, and q_2, respectively. There is one production for each labeled arc, and one λ-production for the final state.

   $S \to bS \,|\, bA \,|\, aB \,|\, cB$
   $A \to aA \,|\, cB$
   $B \to aB \,|\, cB \,|\, bS \,|\, \lambda$

3. [20 points] Construct a minimal DFA equivalent to the NFA shown in Figure 1.

	a	b	c
0	2	01	2
2	2	0	2
01	12	01	2
12	12	0	2





   0 and 01 are equivalent, and 2 and 12 are equivalent. The minimal DFA has two states.

4. [10 points] Give a grammar for the language of all palindromes over $\{a, b\}$.

   There is no regular grammar for this language. Here is a context-free grammar.

   $S \to aSa \,|\, bSb \,|\, a \,|\, b \,|\, \lambda$

5. [15 points] What does it mean to say that an NFA M *accepts* a language L?

If $w \in L$, some computation of M with input w ends at a final state, while if $w \notin L$, there is no computation of M with input w which ends at a final state.

6. [20 points] Prove that $\sqrt{2}$ is irrational.

By contradiction. Assume $\sqrt{2}$ is rational. Then $\sqrt{2} = p/q$ where p, q are integers and $\gcd(p, q) = 1$, *i.e.*, p and q have no common divisor larger than 1. Then:

$$\begin{aligned}
\frac{p}{q} &= \sqrt{2} \\
\frac{p^2}{q^2} &= 2 \\
p^2 &= 2q^2 \\
p^2 \text{ is even} &\Rightarrow p \text{ is even} \\
&\Rightarrow p = 2k \text{ for some integer } k \\
2q^2 &= p^2 \\
&= 4k^2 \\
q^2 &= 2k^2 \\
q^2 \text{ is even} &\Rightarrow q \text{ is even} \\
&\Rightarrow p, q \text{ have a common factor of 2, contradiction.}
\end{aligned}$$