1. True or False. If the question is currently open, write “O” or “Open.”

(i) F Every subset of a regular language is regular.
(ii) T Every DFA is an NFA.
(iii) F Let \( L \) be the language over \( \Sigma = \{a, b\} \) consisting of all strings of the form \( a^m b^n \), for any \( m \) and \( n \). Then \( L \) is a regular language.
(iv) F Let \( L \) be the language over \( \Sigma = \{a, b\} \) consisting of all strings of the form \( a^m b^n \), where \( m \geq n \). Then \( L \) is a regular language.
(v) T The Kleene closure of every regular language is regular.
(vi) T The language consisting of all hexadecimal numerals for positive integers \( n \) such that \( n \mod 13 = 7 \) is regular.
(vii) T The complement of every regular language is regular.
(viii) T The union of any two regular languages is regular.
(ix) T There exists a mathematical proposition that is true, but where no proof of the proposition can exist.
(x) F Every language generated by a grammar is regular.
(xi) O There is a \( \mathcal{P}-\text{time} \) algorithm which decides whether two regular expressions are equivalent.
(xii) T If \( x \) and \( y \) are equivalent regular expressions, there is a \( \mathcal{P}-\text{time} \) proof that \( x \) and \( y \) are equivalent.
(xiii) F The set of all decimal numerals for prime numbers is a regular language.
(xiv) T For any non-deterministic finite automaton, there is always a unique minimal deterministic finite automaton equivalent to it.
(xv) T It can always be decided whether two given regular expressions are equivalent.
(xvi) T The complement, over the binary alphabet, of every regular binary language is regular.
(xvii) T If \( L \) is regular, then \( L^R \) is regular.
(xviii) T Every finite language is regular.
(xix) F The set of all palindromes over the binary alphabet is a regular language.
(xx) T The language of all strings over \( \{a, b\} \) which begin and end with the same symbol is regular.
(xxi) T The intersection of any two regular languages is regular.
(xxii) F The set of all strings which could be expressions in a C++ program is a regular language.
(xxiii) T There is no computer program that decides whether two given C++ programs are equivalent.
2. [20 points] Give a regular grammar for the language accepted by the NFA shown in Figure 1.

Let $S$, $A$, $B$ be the variables corresponding to the states $q_0$, $q_1$, and $q_2$, respectively. There is one production for each labeled arc, and one $\lambda$-production for the final state.

$S \rightarrow bS \mid bA \mid aB \mid cB$

$A \rightarrow aA \mid cB$

$B \rightarrow aB \mid cB \mid bS \mid \lambda$

3. [20 points] Construct a minimal DFA equivalent to the NFA shown in Figure 1.

4. [10 points] Give a grammar for the language of all palindromes over \{a, b\}.

There is no regular grammar for this language. Here is a context-free grammar.

$S \rightarrow aSa \mid bSb \mid a \mid b \mid \lambda$

5. [15 points] What does it mean to say that an NFA $M$ accepts a language $L$?

If $w \in L$, some computation of $M$ with input $w$ ends at a final state, while if $w \notin L$, there is no computation of $M$ with input $w$ which ends at a final state.
6. [20 points] Prove that $\sqrt{2}$ is irrational.

By contradiction. Assume $\sqrt{2}$ is rational. Then $\sqrt{2} = p/q$ where $p, q$ are integers and gcd $(p, q) = 1$, i.e., $p$ and $q$ have no common divisor larger than 1. Then:

\[
\begin{align*}
\frac{p}{q} &= \sqrt{2} \\
\frac{p^2}{q^2} &= 2 \\
p^2 &= 2q^2
\end{align*}
\]

$p^2$ is even $\implies$ $p$ is even

$\implies$ $p = 2k$ for some integer $k$

\[
\begin{align*}
2q^2 &= p^2 \\
&= 4k^2 \\
q^2 &= 2k^2
\end{align*}
\]

$q^2$ is even $\implies$ $q$ is even

$\implies$ $p, q$ have a common factor of 2, contradiction.