Computer Science 456/656 Fall 2019

Answers to Second Examination October 9, 2019

1. True or False. If the question is currently open, write "O" or "Open."

   (i) **O** $\mathcal{NP}$–TIME $= \mathcal{P}$–SPACE.

   (ii) **F** Every context-free language is accepted by some deterministic machine.

   (iii) **O** There is a deterministic machine that accepts SAT in polynomial time.

   (iv) **T** Suppose L_1 and L_2 are $\mathcal{NP}$ languages. If there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, and L_1 is $\mathcal{NP}$–complete, then L_2 must be $\mathcal{NP}$–complete.

   (v) **O** Suppose L_1 and L_2 are $\mathcal{NP}$ languages. If there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, and L_2 is $\mathcal{NP}$–complete, then L_1 must be $\mathcal{NP}$–complete.

   (vi) **T** Let $A = \{a, aa\}$. If L is any decidable language, there is a computable reduction of L to A.

   (vii) **T** Suppose that, next year, someone succeeded in finding a polynomial time algorithm for the knapsack problem. Then we would know that $\mathcal{P} = \mathcal{NP}$.

   (viii) **T** If G is an unambiguous context-free grammar, and if $w \in L(G)$, there must be a unique left-most derivation of w using the grammar G.

   (ix) **T** The regular expression equivalence problem is decidable.

   (x) **T** Every sliding block problem is in the class $\mathcal{P}$–SPACE.

   (xi) **T** The following problem is $\mathcal{NP}$ complete: given a rectangle and a set of polygonal tiles, can the tiles all be placed in the rectangle with no overlap?

   (xii) **T** The concatenation of any two context-free languages is context-free.

   (xiii) **T** The concatenation of any two $\mathcal{NP}$ languages is $\mathcal{NP}$.

   (xiv) **O** If a language is both $\mathcal{NP}$ and CO–$\mathcal{NP}$, it must be $\mathcal{P}$.

   (xv) **F** If L is accepted by some machine, then L must be decidable.

   (xvi) **T** If L is accepted by some machine, and $\overline{L}$ is accepted by some other machine, then L must be decidable.

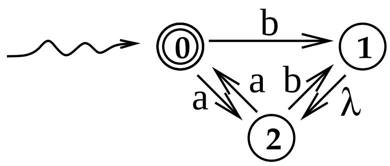
2. [10 points] Give the definition of the partition problem.

Given a set of weighted items, can the set be partitioned into two subsets of equal weight?

3. [10 points] Give the definition of the language SAT.

The set of all satisfiable Boolean expressions. A Boolean expression E is satisfiable if there is an assignment of truth values to the variables of E such that E is true.

4. [20 points] Write a regular expression equivalent to the NFA shown below.



$((a + b)b^*a)^*$

5. In class, we have given two different definitions for the language class $\mathcal{NP}$–TIME.

   (a) [20 points] One of these definitions uses the word *non-deterministic*. Write that definition.

   A language L is in the class $\mathcal{NP}$ if it can be accepted in polynomial time by a non-deterministic machine.

   (b) [20 points] The other definition uses the word *certificate*. Write that definition.

   A language L is in the class $\mathcal{NP}$ if there is a deterministic machine V, called the *verifier*, and for every $w \in L$ there is a string c of polynomial length, called the *certificate*, such that V accepts (w, c) in $\mathcal{P}$ time, and such that for every $w \notin L$, and any string c, V does not accept (w, c).

6. [20 points] State the pumping lemma for regular languages.

For any regular language L there is a positive integer p such that, if $w \in L$ $|w| \leq p$, there exist string x, y, z, such that the following four conditions hold:

   (a) $w = xyz$,

   (b) $|xy| \leq p$,

   (c) $|y| \geq 1$,

   (d) for any integer $i \geq 0$, $xy^iz \in L$.

7. [20 points] The following CF grammar is unambiguous. Give a parse (derivation) tree and a rightmost derivation for the string $x * y - -(x - y * z)$ using that grammar.

$E \to T$
$E \to E + T$
$E \to E - T$
$T \to F$
$T \to T * F$
$F \to -F$
$F \to I$
$F \to (E)$
$I \to x \,|\, y \,|\, z$

$E \Rightarrow E - T \Rightarrow E - F \Rightarrow E - -F \Rightarrow E - -(E) \Rightarrow$
$E - -(E - T) \Rightarrow E - -(E - T * F) \Rightarrow$
$E - -(E - T * I) \Rightarrow E - -(E - T * z) \Rightarrow$
$E - -(E - F * z) \Rightarrow E - -(E - I * z) \Rightarrow$
$E - -(E - y * z) \Rightarrow E - -(T - y * z) \Rightarrow$
$E - -(F - y * z) \Rightarrow E - -(I - y * z) \Rightarrow$
$E - -(x - y * z) \Rightarrow T - -(x - y * z) \Rightarrow$
$T * F - -(x - y * z) \Rightarrow T * I - -(x - y * z) \Rightarrow$
$T * y - -(x - y * z) \Rightarrow F * y - -(x - y * z) \Rightarrow$
$I * y - -(x - y * z) \Rightarrow x * y - -(x - y * z)$