Computer Science 456/656 Fall 2019

Answers to Third Examination November 18, 2019

1. True or False. (5 points each) T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) **T** The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is $\mathcal{P}$–TIME..

   (ii) **O** The problem of whether a given context-**sensitive** grammar generates a given string is in the class $\mathcal{NP}$.

   (iii) **F** The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (iv) **T** Every $\mathcal{NP}$ language is decidable.

   (v) **T** The clique problem is $\mathcal{NP}$-complete.

   (vi) **T** The halting problem is $\mathcal{NP}$–HARD.

   (vii) **T** The union of two $\mathcal{NP}$ languages must be $\mathcal{NP}$.

   (viii) **T** There exists a $\mathcal{P}$-TIME algorithm which finds a maximal independent set in any **acyclic** graph G.

   (ix) **O** $\mathcal{NC} = \mathcal{P}$.

   (x) **T** The traveling salesman problem (TSP) is $\mathcal{NP}$-complete.

   (xi) **T** The language consisting of all satisfiable Boolean expressions is $\mathcal{NP}$-complete.

   (xii) **T** The Boolean Circuit Problem is in $\mathcal{P}$–TIME.

   (xiii) **T** The language consisting of all strings over $\{a, b\}$ which have more a's than b's is context-free.

   (xiv) **T** 2-SAT is $\mathcal{P}$–TIME.

   (xv) **T** Primality, where the input is written in binary, is $\mathcal{P}$-TIME.

   (xvi) **T** multiplication of base 10 numerals is in $\mathcal{NC}$.

   (xvii) **F** The general grammar membership problem is decidable.

   (xviii) **T** EXP–TIME $\subseteq$ EXP–SPACE.

   (xix) **T** The regular expression equivalence problem is decidable.

   (xx) **T** Every sliding block problem is in the class $\mathcal{P}$–SPACE.

   (xxi) **T** The following problem is $\mathcal{NP}$ complete: given a rectangle and a set of polygonal tiles, can the tiles all be placed in the rectangle with no overlap?

(xxii) **T** The Kleene closure of any context-free language is context-free.

(xxiii) **T** The concatenation of any two $\mathcal{NC}$ languages is $\mathcal{NC}$.

(xxiv) **F** If L is accepted by some machine, then L must be decidable.

2. [20 points] Prove that any language that can be enumerated in canonical order by some machine is decidable.

There are two cases. If L is finite, we are done, since every finite language is decidable. If L is infinite, let $w_1, w_2, \ldots$ be the enumeration of L in canonical order. Let P be the following program, which decides whether a given string w is a member of L.

read w
for i = 1 to ∞
  if($w_i = w$) HALT YES
  else if ($w_i > w$) HALT NO

The loop will execute forever, since some subprogram can generated $\{w_i\}$ in canonical order.

3. [20 points] State the pumping lemma for context-free languages.

For any context free language L, there is a positive integer p, such that for any $w \in L$, if $|w| \geq p$, there exist strings u, v, x, y, z such that the following four conditions hold:

(i) $w = uvxyz$

(ii) $|vxy| \leq p$

(iii) $|v| + |y| \geq 1$

(iv) For any integer $i \geq 0$, $uv^ixy^iz \in L$

4. [20 points] Give a $\mathcal{P}$–TIME reduction of the subset sum problem to the partition problem.

An instance $(K, x_1, x_2, \ldots x_n)$ be of the subset sum problem reduces to $(y_1, y_2, \ldots y_n, y_{n+1}, y_{n+1})$ of the partition problem, where $y_i = x_i$ for $i \leq n$, where $y_{n+1} = K$, and where $y_{n+2} = \sum_{i=1}^n x_i - K$

5. [20 points] Prove that the halting problem is undecidable.

By contradiction. Let HALT $= \{\langle M\rangle w \mid M$ halts with input $w\}$. Assume HALT is decidable. Let $L_{diag} = \{\langle M\rangle \mid \langle M\rangle\langle M\rangle \notin \text{HALT}\}$. Since HALT is decidable, L_{diag} is decidable. Let M_{diag} be a machine which accepts L_{diag}. For any machine description $\langle M\rangle$: $\langle M_{diag}\rangle\langle M\rangle \in \text{HALT} \Leftrightarrow \langle M\rangle \in L_{diag}$ by definition of M_{diag}, while $\langle M\rangle\langle M\rangle \in \text{HALT} \Leftrightarrow \langle M\rangle \notin L_{diag}$ by definition of L_{diag}.

By universal instantiation, we can replace $\langle M\rangle$ by $\langle M_{diag}\rangle$ in both of those statements. Thus:

M_{diag} halts with input $\langle M_{diag}\rangle$ implies $\langle M_{diag}\rangle \in L_{diag}$

M_{diag} halts with input $\langle M_{diag}\rangle$ implies $\langle M_{diag}\rangle \notin L_{diag}$

Constradiction. Thus HALT is not decidable.