1. A number is called *rational* if it is the quotient of two integers; otherwise it is called irrational. Prove that \( \sqrt{3} \) is irrational. (Read the proof in the book that \( \sqrt{2} \) is irrational.)

*Proof:* By contradiction. Assume \( \sqrt{3} \) is rational. Then \( \sqrt{3} \) can be written as \( p/q \), where \( p \) and \( q \) are integers. The fraction can be reduced to the lowest terms, meaning that we can assume that the greatest common divisor of \( p \) and \( q \) is 1.

\[
\frac{p}{q} = \sqrt{3}
\]

\[
\frac{p^2}{q^2} = 3
\]

Thus \( p^2 \) is divisible by 3.

Thus \( p \) is divisible by 3.

Write \( p = 3k \) where \( k \) is an integer. Thus

\[
3q^2 = p^2
\]

\[
3q^2 = 9k^2
\]

\[
q^2 = 3k^2
\]

Thus \( q^2 \) is divisible by 3.

Thus \( q \) is divisible by 3.

Thus 3 is a common divisor of \( p \) and \( q \), contradicting the fact that they are relatively prime. \( \blacksquare \)


\[ L(G) = \{(ab)^n : n \geq 0\} \]

Or, work Exercise 15 on page 29 of the sixth edition.

\[ L(G) = \{(aab)^n : n \geq 0\} \]

Work Exercise 13 of page 38 of the fifth edition, which is Exercise 16 on page 29 of the sixth edition.

\[ L(G) = \emptyset \text{ (the empty language)} \]