University of Nevada, Las Vegas Computer Science 456/656 Fall 2019

Answers to Assignment 1: Due Wednesday August 28, 2019

1. A number is called *rational* if it is the quotient of two integers; otherwise it is called irrational. Prove that $\sqrt{3}$ is irrational. (Read the proof in the book that $\sqrt{2}$ is irrational.)

   Proof: By contradiction. Assume $\sqrt{3}$ is rational. Then $\sqrt{3}$ can be written as p/q, where p and q are integers. The fraction can be reduced to the lowest terms, meaning that we can assume that the the greatest common divisor of p and q is 1.

$$\begin{aligned}
\frac{p}{q} &= \sqrt{3} \\
\frac{p^2}{q^2} &= 3 \\
p^2 &= 3q^2 \\
\text{Thus } p^2 \text{ is divisible by 3.} & \\
\text{Thus } p \text{ is divisible by 3.} & \\
\text{Write } p = 3k \text{ where } k \text{ is an integer. Thus} & \\
3q^2 &= p^2 \\
3q^2 &= 9k^2 \\
q^2 &= 3k^2 \\
\text{Thus } q^2 \text{ is divisible by 3.} & \\
\text{Thus } q \text{ is divisible by 3.} &
\end{aligned}$$

   Thus 3 is a common divisor of p and q, contradicting the fact that they are relatively prime. ∎

2. Work Exercise3 12 on page 38 of the fifth edition.

$$L(G) = \{(ab)^n : n \geq 0\}$$

   Or, work Exercide 15 on page 29 of the sixth edition.

$$L(G) = \{(aab)^n : n \geq 0\}$$

   Work Exercide 13 of page 38 of the fifth edition, which is Exercise 16 on page 29 of the sixth edition.

$$L(G) = \emptyset \text{ (the empty language)}$$