University of Nevada, Las Vegas Computer Science 456/656 Fall 2019

Answers to Assignment 4: Due Monday October 7, 2019

1. Work Problem 6 on page 126 of the sixth edition of your textbook, which is problem 5 on page 126 of the fifth edition.

   (a) Not regular.

   (b) Not regular

   (c) Not regular

   (d) Not regular

   (e) Not regular

   (f) Regular. Every integer larger than 1 is either prime or the product of primes.

   (g) Regular. Note that $\{aa, aaa\} \subseteq L$, and $L^* = \{aa, aaa\}^*$ which is regular.

2. Work Problem 22(a) on page 127 of the sixth edition of your textbook, which is problem 19(a) on page 127 of the fifth edition. Justify your answer.

   Regular.

   Recall that a^+ is short for the regular expression aa^*, which desribes the language $\{a^n : n \geq 1\}$. Similarly, $(a+b)^+$ describes the language of non-empty strings over the alphabet $\{a, b\}$. Officially, a^+ is not a regular expression, but unofficially, it is. That is, during an application that uses regular expressions, it is typically permissible to write x^+, where x is any regular expression, instead of x^*x. Thus, u, v, w are arbitrary non-empty strings over $\Sigma = \{a, b\}$.

   We claim that $e = (a+b)^+(aa+bb)(a+b)^+$ is a regular expression for L.

   Proof: Let $L(e)$ be the language desribed by e. If $x \in L(e)$, the either $x = uaav$ or $x = ubbv$ for nontempty strings u, v over Σ. Let $w = a$ in the first case, $w = b$ in the second case. Then $x = uww^Rv$, hence $x \in L$.

   Conversely, let $x \in L$. Then $x = uww^Rv$ where u, v, w are arbitrary non-empty strings over Σ. Then x is either ya or yb, where $y \in \Sigma^*$. Then $x = uyaay^Rv$ or $x = uybby^Rv$, and both uy and y^Rv are non-empty strings over Σ. Thus x is in the language $L(e)$. ∎

   Thus L is regular.

3. We define a GRG, *generalized regular grammar* to consist of generalized productions of one of the following forms:

   $A \to rB$

   $A \to r$

   $A \to B$

   $A \to \lambda$

   where A and B are variables and r is a regular expression. True or false: Every language generated by a GRG is regular.

   True. Each arc of a GRG can be expanded by adding finitely many states and arcs, each with one label.

4. True or false: Every context-free language over the unary alphabet $\{1\}$ is regular. (Hint: Problem 3.)

   True.

5. Consider the "toy" programming language[1] P generated by the following context-free grammar, where the variables are S, L, the terminals are a, w, i, e, b, n and the productions are

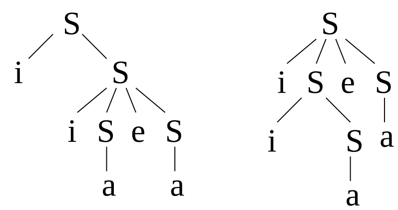
$S \to a$
$S \to wS$
$S \to iS$
$S \to iSeS$
$S \to bLn$
$L \to SL$
$L \to \lambda$

(a) Find a rightmost derivation of $wbaiaewan$.

$S \Rightarrow wS \Rightarrow wbLn \Rightarrow wbSLn \Rightarrow wbSSLn \Rightarrow wbSSn \Rightarrow wbSiSeSn \Rightarrow wbSiSewSn \Rightarrow wbSiSewan \Rightarrow wbSiaewan \Rightarrow wbaiaewan$

(b) Show that the grammar is ambiguous by giving two different parse (derivation) trees for some string.

This is the "dangling else" problem. Pick the string $w = iiaea$



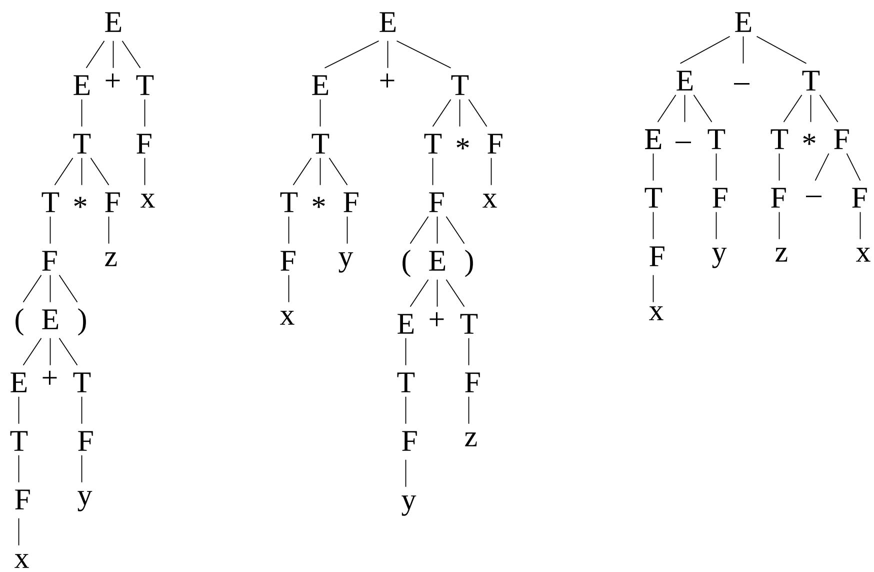
[1]Think of S = statement, a = assignment statement, w = while condition do, i = if condition do, e = else, b = begin or {, n = end or }, L = list of statements.

6. The following grammar unambiguously generates a language of algebraic expressions.

$E \to T$
$E \to E + T$
$E \to E - T$
$T \to F$
$T \to T * F$
$F \to -F$
$F \to I$
$F \to (E)$
$I \to x \,|\, y \,|\, z$

Using that grammar, construct parse (derivation) trees for each of the following strings.

$(x + y) * z + x$
$x * y + (y + z) * x$
$x - y - z * -x$



7. The definition of CNF, *Chomsky normal form*, in our textbook differs from the definition given in some other textbooks. We will use the definition given in our textbook. Under that definition, there is a CNF grammar for any CF language L provided $\lambda \notin L$.

   Give a CNF grammar for the language $L = \{a^n b^m \,:\, n = m + 1\}$.

   $S \to AT$
   $S \to a$
   $T \to SB$
   $A \to a$
   $B \to b$

8. True/False/Open: "Every context-free language is in the class $\mathcal{P}$-TIME." Justify your answer.

   True. Every context-free language is decided by a program which uses the CYK algorithm. That program takes $O(n^3)$ time.