Constructing a Regular Expression from an NFA

To avoid clutter, we do not show any arc whose label is $\emptyset$.

1. In the first example, we replace the multiple final states with a single (new) final state. The label of each arc from an original final state to the new final state is $\lambda$. Note that the start state is no longer final.

We now eliminate States 3 and 2 using Rule 5.

We now eliminate State 1 using Rule 4.

Using Rule 3, we obtain $\lambda + (a+bb)(b+ab+abb)^*a$, the regular expression for the initial NFA.
2. In this example, there is only one final state, but it is the start state. Introduce a new final state and a \( \lambda \) transition from the start state to the final state. Then, eliminate State 1 using Rule 4.

We obtain the regular expression, \((1 + (00 \ast 1))\ast\) by Rule 3. Why didn’t we write \((1 + (00\ast))\ast\lambda\)?

3. Finally, we consider the universal example for three states.

Applying the rules, we obtain an equivalent regular expression

\[(aic^\ast h)^\ast(d + ic^\ast g)(b + fc^\ast g + (e + fc^\ast h)(a + ic^\ast h)^\ast(d + ic^\ast g))^\ast\]