University of Nevada, Las Vegas Computer Science 456/656 Fall 2019

Assignment 8: Due December 6, 2019

Name:____

Finish these problems by the beginning of class, December 6, but do not turn them in.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) ______ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and c's combined. (That is, $w \in L$ if and only if $\#_a(w) > \#_b(w) + \#_c(w)$.) There is some PDA that accepts L.

   (ii) ______ The language $\{a^p \mid p \text{ is prime}\}$ is in the class $\mathcal{P}$-TIME.

   (iii) ______ The dominating set problem is $\mathcal{NP}$-complete.

   (iv) ______ The bin packing problem is $\mathcal{NP}$-complete.

   (v) ______ The minimum spanning tree problem is $\mathcal{NP}$-complete.

   (vi) ______ The general grammar membership problem is $\mathcal{NP}$-complete.

   (vii) ______ The general sliding block problem is $\mathcal{NP}$-complete.

   (viii) ______ If L_1 is an undecidable language and $L_2 \subseteq L_1$ is decidable, then $L_1 \backslash L_2$ must be undecidable.

   (ix) ______ Multiplication of binary numerals is in $\mathcal{NC}$.

   (x) If x and y are integers given as binary numerals, computation of the binary numeral for x^y can always be done in polylogarithmic time with polynomially many processors.

   (xi) ______ Every context-sensitive language is in $\mathcal{P}$.

   (xii) ______ Every language generated by a general grammar is recursively enumerable.

   (xiii) It is decidable whether a context free gammar with terminal alphabet Σ generates Σ^*.

   (xiv) ______ If G_1 and G_2 are context-free grammars and $L(G_1) \neq L(G_2)$, there is a proof that $L(G_1) \neq L(G_2)$.

   (xv) ______ The Post correspondence problem is undecidable.

   (xvi) ______ The intersection of two recursively enumerable languages must be recursively enumerable.

   (xvii) ______ If x is a real number, and if the set of fractions whose valuea are less than x is decidable, then there must be a recursive function D where $D(n)$ is the n^{th} digit after the decimal point in the decimal expansion of x.

   (xviii) ______ There exists an algorithm which finds the prime factors of any positive integer, where the input is given as a binary numeral.

2. Which of these problems (not necessarily 0/1 problems) are **known** to be workable by polynomially many processors in polylogarithmic time? Answer **Y** for yes, **N** for no. For example, if a problem is known to be in $\mathcal{P}$–TIME but not known to be in $\mathcal{NC}$, the answer is **N**, since it is not known whether $\mathcal{P} = \mathcal{NC}$.

   (a) ______ Compute the binary numeral for x^y, where x and y are given as binary numerals.

   (b) ______ Is a given array sorted?

   (c) ______ The Boolean circuit problem.

   (d) ______ Sorting an array.

   (e) ______ Determining whether a given string is accepted by a given NFA.

   (f) ______ Finding a minimal spanning tree of a weighted graph. (Kruskal's algorithm and Primm's algorithm solve this problem in polynomial time.)

   (g) ______ Finding an optimal prefix-free binary code for a weighted alphabet. (Huffman's algorithm solves this problem in polynomial time.)

   (h) ______ Computing the first n Fibonacci numbers.

   (i) ______ Connectivity. Given a graph (not digraph) G, is G connected?

   (j) ______ Boolean satisfiability. (Otherwise known as SAT.)

   (k) ______ 2-SAT.

   (l) ______ Context-free membership.

   (m) ______ Context-sensitive membership.

3. Which of these functions are recursive? Answer **Y** if the function is recursive, **N** if it is not recursive.

   (a) ______ The Ackermann function.

   (b) ______ The Busy Beaver function.

   (c) ______ The number of members of L which have length exactly n, where L is some R.E. but undecidable language over the binary alphabet $\Sigma = \{0, 1\}$.

   (d) ______ The n^{th} decimal digit of π.

   (e) ______ The n^{th} decimal digit of Chaitin's constant.

   (f) ______ The n^{th} decimal digit of a given real algebraic number, *i.e.* a real number which is a root of a given polynomial with integral coefficients.

4. Give an NFA with six states whose equivalent minimal DFA has 64 states (including the dead state).

5. Fill each the blank with one of the following three words: preorder, postorder, level order.

   (a) A recursive-descent parser determines the internal nodes of the parse tree in ________________________.

   (b) An LALR parser determines the internal nodes of the parse tree in ________________________.

6. A context-free grammar is *linear* if the right side of each production has at most one variable. Find a linear grammar which generates a language which is not regular.

7. Give an example of an unambiguous context-free grammar G, such that $L(G)$ is not accepted by any DPDA.

8. Give an example of an ambiguous context-free grammar G such that there is a DPDA which finds a parse tree of every $w \in L(G)$.

9. Given an example of an undecidable $\mathcal{NP}$-complete problem.

10. Give an example of two undecidable languages whose intersection is decidable.

11. Find two undecidable languages whose union is decidable.

12. Prove that L^R is regular for any regular language L. (Hint: NFA)

13. Give a polynomial time reduction of 4-SAT to 3-SAT.

14. Give a polynomial time reduction of 3-SAT to 2-SAT.

15. Construct a PDA which accepts the algebraic language whose grammar is given below, where E is the start symbol. The grammar is unambiguous. This is a rather hard problem. In the compiler class, it gets much worse. (If I leave out the second production, the problem is much easier.)

$E \to E - T$
$E \to T$
$T \to (E)$
$T \to x$

16. Give a proof that the factoring problem is both $\mathcal{NP}$ and co-$\mathcal{NP}$. (The factoring problem is, given a binary numeral for a number n and another binary numeral for a number a, determine whether there is divisor d of n which is less than a but more than 1.)

17. Prove that if $\mathcal{NP} = \mathcal{P}$, no trapdoor function exists.

18. Find a general grammar which generates $\left\{w \in \{a,b\}^* \mid \#_a(w) + \#_b(w) = 2^n, n \geq 0\right\}$ (I can do it with 5 variables and 8 productions. Can you do better?)