University of Nevada, Las Vegas Computer Science 456/656 Fall 2020

Assignment 2: Due Friday September 25, 2020

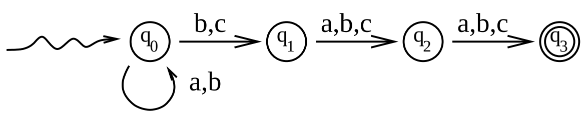
Name:____

You are permitted to work in groups, get help from others, read books, and use the internet. Your answers must be written in a pdf file and emailed to the graudate assistant, Shekhar Singh `shekhar.singh@unlv.edu` by 23:59 September 25. Your file must not exceed 5 megabytes, and must print out to at most 4 pages.

Theorem 1 *If a language L is accepted by an NFA M with n states, then n is a pumping length of L.*

Proof: Suppose $w \in L$ and $|w| \geq n$. Let $w = a_1 a_2 \cdots a_m$ for $m \geq n$ and $a_i \in \Sigma$ for all i. Let $\sigma = q_0 q_1 \ldots q_m$ be a computaion of M, which accepts w. Since $m + 1 > n$, there exist $0 \leq j < k \leq m$ such that $q_j = q_k$, by the pigeonhole principle.[1] If $j = 0$, let $x = \lambda$, otherwise let $x = a_{j+1} \cdots a_i$. Let $y = a_{i+1} \cdots a_j$. If $j = 0$, let $z = \lambda$, otherwise let $z = a_{j+1} \cdots a_m$. Note that $xyz = w$. Let σ_x be the computational path $q_0 \cdots q_{i-1} q_i$, let σ_z be the computational path $q_j q_{j+1} \cdots q_m$, and let σ_y be the path $q_i q_{i+1} \cdots q_{j-1} q_j$, which is a loop since $q_i = q_j$. For any $i \geq 0$, $xy^i z \in L$ since it is accepted by M with the computation $\sigma_x \sigma_y^i \sigma_z$. ∎

1. Let L be the regular language accepted by the following NFA.



   (a) By the theorem, 4 is a pumping length of L. Note that $bbac \in L$.Find a pumpable substring of $bbac$

   (b) L contains strings of length 3. Show that 3 is not a pumping length for L.

2. Let $L = \{ab^n c^m : n \geq m\}$. Use the pumping lemma to prove that L is not regular.

3. The language $L = \{a^n b^m c^m d^n : n, m \geq 0\}$ is context-free, and has pumping length 2. Let $w = aabbccdd$. Find the strings u, v, x, y, z given by the pumping lemma for context-free languages.

4. Give a diagram for a PDA which accepts the context-free language given in Problem 3.

[1] If n holes hold $n + 1$ pigeons, then there must be at least one hole that has at least two pigeons.