University of Nevada, Las Vegas Computer Science 456/656 Fall 2020
Assignment 4: Due Friday November 6, 2020

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet. Your answers must be written in a pdf file and emailed to the graudate assistant, Shekhar Singh `shekhar.singh@unlv.edu` by 23:59 November 6. Your file must not exceed 10 megabytes, and must print out to at most 8 pages.

1. Suppose L is the regular language accepted by an NFA M with p states. Prove that L has [regular] pumping length p.

2. Find an NFA with three states whose equivalent minimal DFA has eight states.

3. Given alphabets Σ_1 and Σ_2, a *homomorphism* from Σ_1 to Σ_2 is a function $h : \Sigma_1 \to \Sigma_2^*$. For example, any Huffman code on an alphabet Σ is a homomorphism from Σ to the binary alphabet. If $h : \Sigma_1 \to \Sigma_2^*$ is a homomorphism and $w \in \Sigma_1^*$, we define $h(w)$ to be the string obtained by replacing each symbol x of w by $h(x)$. (For example, if h is the huffman code $a \mapsto 10$, $b \mapsto 1110$, $c \mapsto 1111$, $d \mapsto 110$, $e \mapsto 0$, then $h(bed) = 11100110$.) If $L \subseteq \Sigma_1^*$, let $h(L) = \{h(w) : w \in L\}$. If L is regular, is it always true that $h(L)$ is regular? Why do you believe that?

4. What is the minimum [regular] pumping length of the language of all decimal numerals for multiples of three? Exact answer, please. Warning: The empty string is not a decimal numeral.

5. We say that a set D of vertices of a graph G *dominates* G if every vertex of G is adjacent to some member of D. The Dominating Set problem is, given a graph G and a number k, does G have a dominationg set of order (that is, size) k? From what we've covered so far in class, we know that SAT, 3-SAT, Independent set, Partition, Subset Sum, and regular expression equivalence are all $\mathcal{NP}$-complete. Using that knowledge, prove that the dominating set problem is $\mathcal{NP}$-complete

6. What is the minimum [context-free] pumping length of $L = L_{\text{Dyck}}\backslash\{\lambda\}$?

7. Find a context-free language which is not accepted by any DPDA. Justify your answer.

8. Consider the following problems:

   (a) The furniture placement problem. Given a room of certain dimensions, and given a set of pieces of furniture, it is possible to place all the furniture into the room? You are permitted to lower furniture through the ceiling with a crane.

   (b) The furniture moving problem. Given a room of certain dimensions, with a given door, and given a set of pieces of furniture, is it possible to move all the furniture into the room through the door?

   Refer to the Euler diagram of complexity classes handed out earlier, what is the smallest of those complexity classes that is known to contain the furniture placement problem, and what is the smallest of those complexity class that is known to contain the furniture moving problem? I am not asking for proofs.

9. Let $L = \{a^n b^n c^n : n \geq 1\}$. Let G be the context-sensitive grammar with productions:
   $S \to abc$
   $ab \to aaAbb$
   $Ab \to bA$
   $Ac \to cc$
   Does G generate L? Justify your answer.

10. Prove that the context-free grammar equivalence problem is co-RE.

11. Is every decidable language context-sensitive? You may give an answer you find on the internet. If you do that, give the url.