Computer Science 456/656 Fall 2020

Practice for First Examination September 14, 2020

The entire practice examination is 310 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (a) _____ Every subset of a regular language is regular.

   (b) _____ The Dyck language is regular.

   (c) _____ If a language L is generated by some context-free grammar, then L is accepted by some PDA.

   (d) _____ If L is a language accepted by some PDA, then L is generated by some context-free grammar.

   (e) _____ The Kleene closure of every context-free language is context-free.

   (f) _____ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   (g) _____ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   (h) _____ There is a PDA that accepts the language consisting of all C++ programs.

   (i) _____ Let L be the language over $\Sigma = \{a, b, c\}$ consisting of all strings of the form $a^n b^n c^n$, where $n \geq 0$. Then L is a context-free language.

   (j) _____ Let L be the language over $\Sigma = \{a, b, c, d\}$ consisting of all strings of the form $a^n b^m c^p d^q$, where $0 \leq n \leq q$ and $0 \leq m \leq p$. Then L is a context-free language.

   (k) _____ The intersection of any two context-free languages is context-free.

   (l) _____ The union of any two context-free languages is context-free.

   (m) _____ The language $\{a^m b c^n \ : \ 0 \leq m \leq n\}$ is accepted by some DPDA.

   (n) _____ The membership problem for context-free languages is decidable.

   (o) _____ The equivalence problem for context-free grammars is decidable.

   (p) _____ Every DFA is an NFA.

   (q) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, for any m and n. Then L is a regular language.

   (r) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a regular language.

   (s) _____ Every regular language is context-free.

   (t) _____ The Kleene closure of every regular language is regular.

   (u) _____ The language consisting of all hexadecimal numerals for positive integers n such that $n \ \% \ 13 = 7$ is regular.

   (v) _____ The complement of every regular language is regular.

   (w) _____ The union of any two regular languages is regular.

(x) _____ Every NFA is a DFA.

(y) _____ The intersection of any two regular languages is regular.

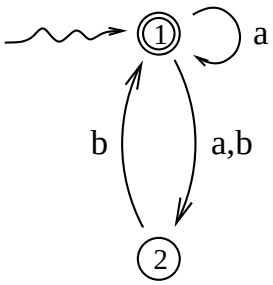
(z) _____ There exists a mathematical proposition that is true, but where no proof of the proposition can exist.

2. [20 points] Let L be the language consisting of all strings over the binary alphabet whose last three symbols are '`010`.' Draw an NFA with four states which accepts L.

3. [20 points] Describe the language L generated by the following context-free grammar where $\{a, b\}$ is the set of terminals, $\{S\}$ is the set of variables, S is the start symbol, and the productions are as follows:

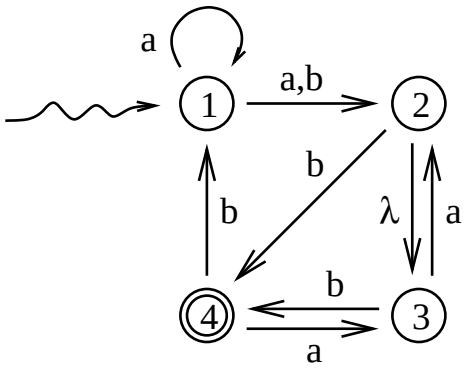
   1. $S \to aSb$
   2. $S \to aS$
   3. $S \to \varepsilon$

4. [20 points] Write a regular expression for the language accepted by the NFA shown below.



5. [20 points] Let L be the language consisting of all strings over $\{a, b\}$ which do not contain the substring aab. Write a regular expression for L and draw a minimal DFA which accepts L. (Hint: 3 states.)

6. [40 points] Draw a state diagram for a minimal DFA equivalent to the NFA shown below. Partial credit if you get the first steps correct. λ-transitions are discussed in Section 2.2 of your textbook, and an NFA with a *lambda*-transtion is given in Figure 2.9. https://www.youtube.com/watch?v=4bjqVsoy6bA is a youtube video that you might want to watch.



7. [5 points] The _____ algorithm decides whether a given string is a member of a given context-free language.

8. [5 points] _____ has an unambiguous context-free grammar, but is not accepted by any DPDA.

9. [20 points] Let G be the context-free grammar given below.

   $S \to a$

   $S \to wS$

   $S \to iS$

   $S \to iSeS$

   Prove that G is ambiguous by writing two different parse trees for the string $iwiaea$.

10. [30 points] Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

(a) $S \to a$

(b) $E \to a$

(c) $S \to LA$

(d) $E \to LA$

(e) $L \to ($

(f) $A \to ER$

(g) $R \to)$

(h) $S \to PE$

(i) $E \to PE$

(j) $S \to EE$

(k) $E \to EE$

(l) $P \to EQ$

(m) $Q \to +$

Use the CYK algorithm to prove that the string $a(a+a)$ is a member of L. Use the figure below for your work. You might want to watch the Youtube video https://www.youtube.com/watch?v=I5E3uU15sjQ.