# Computer Science 456/656 Fall 2020

Answers to Practice for First Examination September 14, 2020

The entire practice examination is 315 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

    (a) **F** Every subset of a regular language is regular.

    (b) **F** The Dyck language is regular.

    (c) **T** If a language L is generated by some context-free grammar, then L is accepted by some PDA.

    (d) **T** If L is a language accepted by some PDA, then L is generated by some context-free grammar.

    (e) **T** The Kleene closure of every context-free language is context-free.

    (f) **F** If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

    (g) **F** If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

    (h) **F** There is a PDA that accepts the language consisting of all C++ programs.

    (i) **F** Let L be the language over $\Sigma = \{a, b, c\}$ consisting of all strings of the form $a^n b^n c^n$, where $n \geq 0$. Then L is a context-free language.

    (j) **T** Let L be the language over $\Sigma = \{a, b, c, d\}$ consisting of all strings of the form $a^n b^m c^p d^q$, where $0 \leq n \leq q$ and $0 \leq m \leq p$. Then L is a context-free language.

    (j) **F** Let L be the language over $\Sigma = \{a, b, c, d\}$ consisting of all strings of the form $a^n b^m c^p d^q$, where $0 \leq n \leq p$ and $0 \leq m \leq q$. Then L is a context-free language.

    (k) **F** The intersection of any two context-free languages is context-free.

    (l) **T** The union of any two context-free languages is context-free.

    (m) **T** The language $\{a^m b c^n : 0 \leq m \leq n\}$ is accepted by some DPDA.

    (n) **T** The membership problem for context-free languages is decidable.

    (o) **F** The equivalence problem for context-free grammars is decidable.

    (p) **T** Every DFA is an NFA.

    (q) **T** Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, for any m and n. Then L is a regular language.

    (r) **F** Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a regular language.

    (s) **T** Every regular language is context-free.

    (t) **T** The Kleene closure of every regular language is regular.

    (u) **T** The language consisting of all hexadecimal numerals for positive integers n such that $n \% 13 = 7$ is regular.

(v) **T** The complement of every regular language is regular.

(w) **T** The union of any two regular languages is regular.

(x) **F** Every NFA is a DFA.

(y) **T** The intersection of any two regular languages is regular.

(z) **T** There exists a mathematical proposition that is true, but where no proof of the proposition can exist.
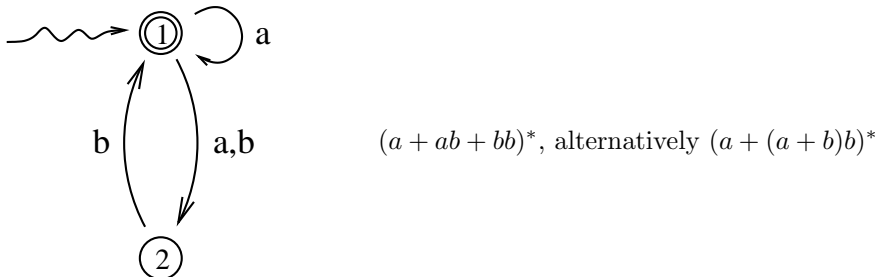
2. [20 points] Let L be the language consisting of all strings over the binary alphabet whose last three symbols are '010.' Draw an NFA with four states which accepts L.
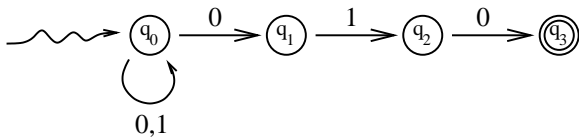

3. [20 points] Describe the language L generated by the following context-free grammar where $\{a, b\}$ is the set of terminals, $\{S\}$ is the set of variables, S is the start symbol, and the productions are as follows:

1. $S \to aSb$

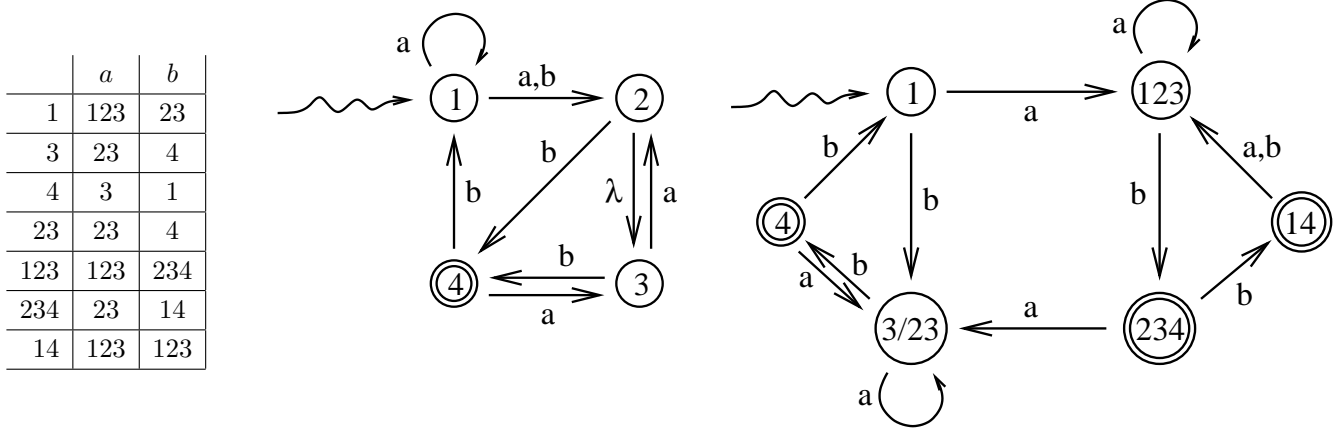2. $S \to aS$ $\{a^n b^m : m \leq n\}$

3. $S \to \lambda$

4. [20 points] Write a regular expression for the language accepted by the NFA shown below.

$(a + ab + bb)^*$, alternatively $(a + (a + b)b)^*$

5. [20 points] Let L be the language consisting of all strings over $\{a, b\}$ which do not contain the substring aab. Write a regular expression for L and draw a minimal DFA which accepts L. (Hint: 3 states, if you don't count the dead state.)

$(ab + b)^* a^*$
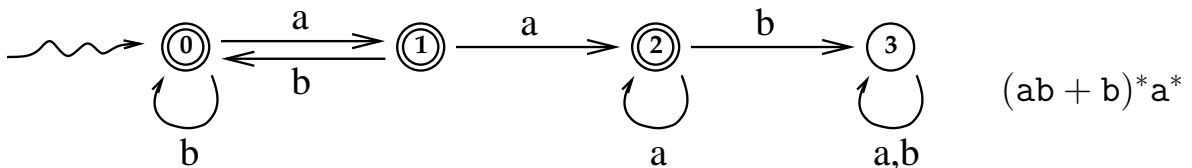
6. [40 points] Draw a state diagram for a minimal DFA equivalent to the NFA shown below. Partial credit if you get the first steps correct. λ-transitions are discussed in Section 2.2 of your textbook, and an NFA with a *lambda*-transtion is given in Figure 2.9. https://www.youtube.com/watch?v=4bjqVsoy6bA is a youtube video that you might want to watch.

|	a	b
1	123	23
3	23	4
4	3	1
23	23	4
123	123	234
234	23	14
14	123	123

7. [5 points] The **CYK** algorithm decides whether a given string is a member of a given context-free language.

8. [5 points] _____ has an un-ambiguous context-free grammar, but is not accepted by any DPDA.

   There are many correct answers, but the only one we've discussed in class is the language of all palindromes over the alphabet $\{a, b\}$.

9. [20 points] Let G be the context-free grammar given below.

   $S \rightarrow a$

   $S \rightarrow wS$

   $S \rightarrow iS$

   $S \rightarrow iSeS$

   Prove that G is ambiguous by writing two different parse trees for the string *iwiaea*.
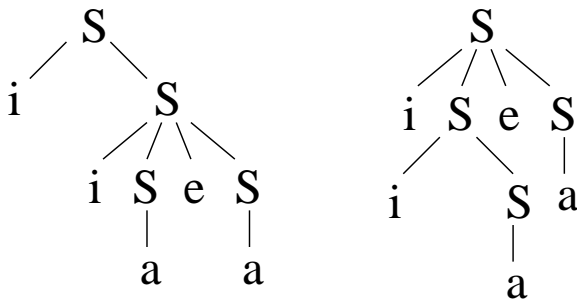
10. [30 points] Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

   (a) $S \to a$

   (b) $E \to a$

   (c) $S \to LA$

   (d) $E \to LA$

   (e) $L \to ($

   (f) $A \to ER$

   (g) $R \to)$

   (h) $S \to PE$

   (i) $E \to PE$

   (j) $S \to EE$

   (k) $E \to EE$

   (l) $P \to EQ$

   (m) $Q \to +$

   Use the CYK algorithm to prove that the string $a(a+a)$ is a member of L. Use the figure below for your work. You might want to watch the Youtube video https://www.youtube.com/watch?v=I5E3uU15sjQ.