Computer Science 456/656 Fall 2018 Practice Examination October 17, 2018

The entire examination is 451 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

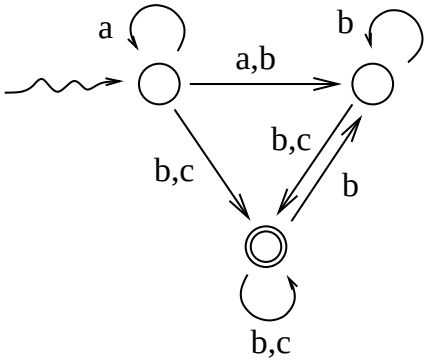
   Review the 3 page list of true false questions on the web page.

2. [20 points] State the pumping lemma for regular languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

3. [20 points] State the pumping lemma for context-free languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

4. [25 points] Draw an NFA with five states which accepts the language described by the regular expression $(0+1)^*0(0+1)(0+1)(0+1)$

5. [25 points] Write a regular expression for the language accepted by the following NFA. If your answer is unnecessarily long by a wide margin, I might mark it wrong even if it's right.



Find a Regular Expression

6. [20 points] Let G be the context-free grammar given below.

$S \to a$

$S \to wS$

$S \to iS$

$S \to iSeS$

Prove that G is ambiguous by writing two different **leftmost** derivations for the string *iwiaea*. [If you simply show two different parse trees, you are not following instructions.]

7. [30 points] Design a PDA that accepts the language $L = \{a^n b c^n \ : \ n \geq 0\}$.

8. [30 points] Give a context-free grammar for the language of all strings over $\{0, 1\}$ of the form $0^m 1^n$ where $n \neq m$.

9. [30 points] The following context-free grammar G is ambiguous. Give an equivalent unambiguous grammar.

1. $E \ \to E \ + \ E$
2. $E \ \to E \ - \ E$
3. $E \ \to E \ * \ E$
4. $E \ \to - \ E$
5. $E \ \to (\ E \ )$
6. $E \ \to a$
7. $E \ \to b$
8. $E \ \to c$

10. [30 points] Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

(a) $S \to a$

(b) $E \to a$

(c) $S \to LA$

(d) $E \to LA$

(e) $L \to ($

(f) $A \to ER$

(g) $R \to)$

(h) $S \to PE$

(i) $E \to PE$

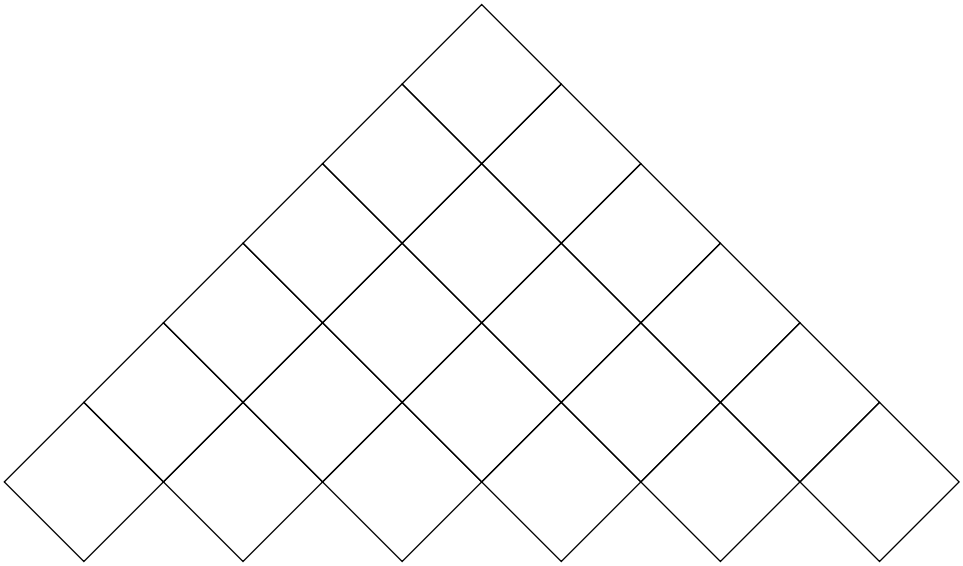
(j) $S \to EE$

(k) $E \to EE$

(l) $P \to EQ$

(m) $Q \to +$

Use the CYK algorithm to prove that the string $a(a + a)$ is a member of L. Use the figure below for your work.



11. [30 points] Consider the NFA whose transition diagram is in Figure 1 below. where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

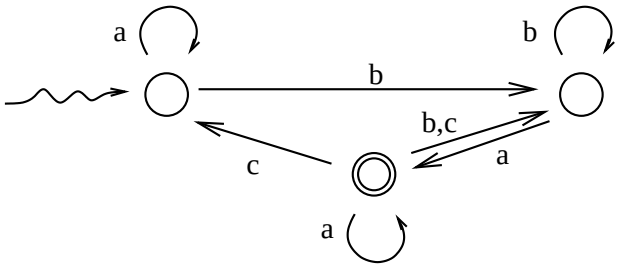


Figure 1: Find a minimal DFA equivalent to this NFA

12. [30 points] Let $L = \left\{w \in \{a, b\}^* \mid \#_a(w) = 2\#_b(w)\right\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, *aaababaaabba* $\in L$, because that string has the twice as many a's as b's. Give a context-free grammar for L. Your grammar may be ambiguous.

    Fill in the blank. If L_1 is undecidable and if R is a reduction of L_1 to L_2 and if R is _____, then L_2 is undecidable.

13. [20 points] State the pumping lemma for context-free languages.

14. [20 points] Use the pumping lemma for context-free languages to prove that the language $L = \{a^n b^n c^n\}$ is not context-free.

15. [20 points] Prove that a language is recursively enumerable if and only if it is accepted by some machine.

16. [20 points] Prove that the halting problem is undecidable.

17. [20 points] Prove that the context-free grammar equivalence problem is co-RE.

18. [20 points] Give a definition of each of these $\mathcal{NP}$-complete languages/problems.

    (a) SAT

    (b) 3-SAT

    (c) Independent Set

    (d) Subset Sum

    (e) Partition

    The following problems deal with material I hope to cover before the examination.

19. [20 points] Give a general (unrestricted) grammar for the language consisting of all string of 1's of length a power of 2, that is, $\left\{1^{2^n}\right\}$

20. [20 points] Give one of these polynomial time reductions (your choice).

(a) 3-SAT to Independent Set.

(b) Independent Set to Subset Sum

(c) Subset Sum to Partition