Computer Science 456/656 Fall 2020 Practice Examination October 7, 2018

Some of these problems are copied from the practice2. Those deal with issues that were not on Test 2.

1. All the problems of Assignment 3.

2. All the problems of Assignment 4.

3. True or False.T = true, F = false, and O = open, meaning that the answer is not known to science at this time. The first 42 questions should be familiar to you.

   (i) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (ii) _____ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

   (iii) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (iv) _____ The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.

   (v) _____ The intersection of any three regular languages is context-free.

   (vi) _____ If L is a context-free language over an alphabet with just one symbol, then L is regular.

   (vii) _____ There is a deterministic parser for any context-free grammar.

   (viii) _____ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

   (ix) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (x) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (xi) _____ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

   (xii) _____ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

   (xiii) _____ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   (xiv) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$-TIME.

   (xv) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (xvi) _____ Every undecidable problem is $\mathcal{NP}$-complete.

   (xvii) _____ Every problem that can be mathematically defined has an algorithmic solution.

(xviii) _____ The intersection of two undecidable languages is always undecidable.

(xix) _____ Every $\mathcal{NP}$ language is decidable.

(xx) _____ The intersection of two $\mathcal{NP}$ languages must be $\mathcal{NP}$.

(xxi) _____ The intersection of two $\mathcal{NP}$-complete languages must be $\mathcal{NP}$-complete.

(xxii) _____ $\mathcal{NC} = \mathcal{P}$.

(xxiii) _____ $\mathcal{P} = \mathcal{NP}$.

(xxiv) _____ $\mathcal{NP} = \mathcal{P}$-SPACE

(xxv) _____ $\mathcal{P}$-SPACE = EXP-TIME

(xxvi) _____ EXP-TIME = EXP-SPACE

(xxvii) _____ The traveling salesman problem (TSP) is $\mathcal{NP}$-complete.

(xxviii) _____ The knapsack problem is $\mathcal{NP}$-complete.

(xxix) _____ The language consisting of all satisfiable Boolean expressions is $\mathcal{NP}$-complete.

(xxx) _____ The Boolean Circuit Problem is in $\mathcal{P}$.

(xxxi) _____ The Boolean Circuit Problem is in $\mathcal{NC}$.

(xxxii) _____ If L_1 and L_2 are undecidable langugages, there must be a recursive reduction of L_1 to L_2.

(xxxiii) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

(xxxiv) _____ The language consisting of all strings over $\{a, b\}$ which have more a's than b's is context-free.

(xxxv) _____ 2-SAT is $\mathcal{P}$-TIME.

(xxxvi) _____ 3-SAT is $\mathcal{P}$-TIME.

(xxxvii) _____ Primality is $\mathcal{P}$-TIME.

(xxxviii) _____ There is a $\mathcal{P}$-TIME reduction of the halting problem to 3-SAT.

(xxxix) _____ Every context-free language is in $\mathcal{P}$.

(xl) _____ Every context-free language is in $\mathcal{NC}$.

(xli) _____ Addition of binary numerals is in $\mathcal{NC}$.

(xlii) _____ Every context-sensitive language is in $\mathcal{P}$.

(xliii) _____ Every language generated by a general grammar is recursive.

(xliv) _____ The problem of whether two given context-free grammars generate the same language is decidable.

(xlv) _____ The language of all fractions (using base 10 numeration) whose values are less than π is decidable. (A *fraction* is a string. "314/100" is in the language, but "22/7" is not.)

(xlvi) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary ("caveman") numeral.

(xlvii) _____ For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(xlviii) _____ For any two languages L_1 and L_2, if L_2 is undecidable and there is a recursive reduction of L_1 to L_2, then L_1 must be undecidable.

(xlix) As you may have learned, there is a formal language which can be used to write any mathematical proposition as well as any proof of any mathematical proposition, and an algorithm exists that can check the correctness of such a proof. In 1978, Jack Milnor https://en.wikipedia.org/wiki/John_Milnor told me that in the future no proof will be accepted unless it can be verified by a computer.
_____ If P is a mathematical proposition that can be written using a string of length n, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

(l) _____ If L is any $\mathcal{NP}$ language, there must be a $\mathcal{P}$–TIME reduction of the partition problem to L.

(li) _____ Every bounded function is recursive.

(lii) _____ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(liii) _____ Recall that if $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$. Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

(liv) _____ Every language is enumerable.

(lv) _____ If a language L is undecidable, then there can be no machine that enumerates L.

(lvi) _____ There exists a mathematical proposition that can be neither proved nor disproved.

(lvii) _____ There is a non-recursive function which grows faster than any recursive function.

(lviii) _____ There exists a machine[1] that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(lix) _____ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(lx) _____ **Rush Hour**, the puzzle sold in game stores everywhere, generalized to a board of arbitrary size, is $\mathcal{NP}$–complete.

(lxi) _____ There is a polynomial time algorithm which determines whether any two regular expressions are equivalent.

[1] As always in automata theory, "machine" means abstract machine, a mathematical object whose memory and running time are **not** constrained by the size and lifetime of the known (or unknown) universe, or any other physical laws. If we want to discuss the kind of machine that exists (or could exist) physically, we call it a "physical machine."

(lxii) _____ If two regular expressions are equivalent, there is a polynomial time proof that they are equivalent.

(lxiii) _____ $\mathcal{P}$–TIME = EXP–TIME.

(lxiv) _____ $\mathcal{P}$–SPACE = EXP–SPACE.

(lxv) _____ Multiplication of integers is in $\mathcal{NC}$.

(lxvi) _____ Every context-free language is in $\mathcal{NC}$.

(lxvii) _____ Every context-sensitive language is in $\mathcal{P}$–TIME.

(lxviii) _____ Any language generated by an unrestricted [general] grammar is recursively enumerable.

(lxix) If there is a polynomial time reduction of L_1 to L_2, and L_1 is $\mathcal{P}$-complete and L_2 is $\mathcal{P}$, then L_2 must be $\mathcal{P}$-complete.

For the following four questions, let Σ_1 and Σ_2 be alphabets, and $h : \Sigma_1 \to \Sigma_2^*$, a homomorphism.

(lxx) _____ If $L \subseteq \Sigma_1^*$ is regular, then $h(L)$ is regular.

(lxxi) _____ If $L \subseteq \Sigma_2^*$ is regular, then $h^{-1}(L)$, defined to be $\{w \in \Sigma_1^* : h(w) \in L\}$, is regular.

(lxxii) _____ If $L \subseteq \Sigma_1^*$ is context-free, then $h(L)$ is context-free.

(lxxiii) _____ If $L \subseteq \Sigma_1^*$ is decidable, then $h(L)$ is decidable.

4. State the pumping lemma for regular languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

5. State the pumping lemma for context-free languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

6. Give a context-free grammar for the language of all strings over $\{a, b, c\}$ of the form $a^n b c^n$ for $n \geq 0$.

7. The following context-free grammar G is ambiguous. Give an equivalent unambiguous grammar.

    1. $E \to E + E$
    2. $E \to E - E$
    3. $E \to E * E$
    4. $E \to - E$
    5. $E \to (E)$
    6. $E \to a$
    7. $E \to b$
    8. $E \to c$

8. Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

   (i) $S \to a$

   (ii) $E \to a$

   (iii) $S \to LA$

   (iv) $E \to LA$

   (v) $L \to ($

   (vi) $A \to ER$

   (vii) $R \to)$

   (viii) $S \to PE$

   (ix) $E \to PE$

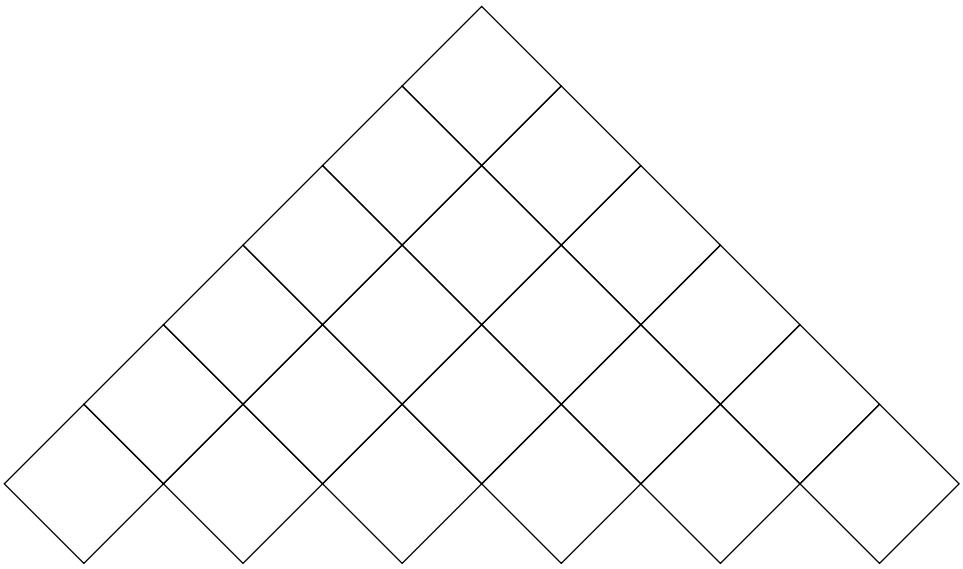
   (x) $S \to EE$

   (xi) $E \to EE$

   (xii) $P \to EQ$

   (xiii) $Q \to +$

   Use the CYK algorithm to prove that the string $a(a + a)$ is a member of L. Use the figure below for your work.



9. Consider the NFA whose transition diagram is in Figure 1 below. where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

10. Let $L = \{w \in \{a, b\}^* \mid \#_a(w) = 2\#_b(w)\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, *aaababaaabba* $\in L$, because that string has the twice as many a's as b's. Give a context-free grammar for L. Your grammar may be ambiguous.

11. Give a conext-sensitive grammar for $\{a^n b^n c^n : n \geq 1\}$

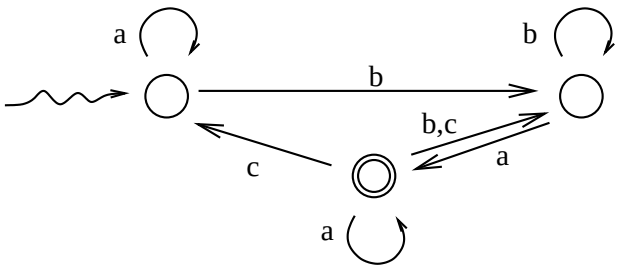


Figure 1: Find a minimal DFA equivalent to this NFA

12. $\mathcal{NP}$-complete languages/problems.

    (i) 3-SAT

    (ii) Independent Set

    (iii) Dominating Set

    (iv) Subset Sum

    (v) Partition

13. Give a general (unrestricted) grammar for the language consisting of all string of 1's of length a power of 2, that is, $\left\{1^{2^n}\right\}$

14. State the Church-Turing thesis. Why it is important?

15. $\mathcal{NP}$-completelness.