University of Nevada, Las Vegas Computer Science 456/656 Fall 2021

Assignment 3. Due 11:30 AM Monday September 20, 2021

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet.

Do not hand this homework in. It will not be graded, but I will go over it in class on September 20.

1. Starting on page 138 of the sixth edition of your textbook or on page 137 of the fifth edition, work problems 9(b), 9(e), 12(c),and 15.

2. I will give the proof by contradiction that $\sqrt{2}$ is irrational in class. Prove by contradiction that $\sqrt{3}$ is irrational.

3. You have probably seen the easy (high school) proof that the sum of the first n positive integers is $\frac{n(n+1)}{2}$. That formula was made famous by Johann Carl Friedrich Gauss, who came up with the proof at age 10 to work a class asignment in his head in less than a minute while the other pupils were busy for an hour. He was not the first to find the formula, which was well-known at the time. The teacher knew it, but didn't tell the kids. There is also a proof by induction, which I will give in class.

   Use induction to prove that the sum of the squares of the first n positive integers is $\frac{n(n+1)(2n+1)}{6}$. What is the sum of the cubes of the first n positive integers? Look it up.

The context-free grammar G_1 given below generates an algebraic language L, where the strings of L are C++ expressions with variables and operators. There are only three C++ variables, x, y, and z, and there are only four operations, addition, subtraction, negation, and multiplication. The start symbol is E (for "expression") instead of the usual S, and the alphabet of L is $\{x, y, z, +, -, *, (,)\}$

(a) Prove that G_1 is ambiguous by giving two different left-most derivations of the string $x + y * z$ and the corresponding derivation trees.

    1. $E \rightarrow E + E$
    2. $E \rightarrow E - E$
    3. $E \rightarrow E * E$
    4. $E \rightarrow -E$
    5. $E \rightarrow (E)$
    6. $E \rightarrow x$
    7. $E \rightarrow y$
    8. $E \rightarrow z$

(b) When I input a certain string $w \in L$ into my top-down parser, it outputs 352674513687, the encoding of a left-most derivation of w. Find w.

(c) The grammar G_1 is does not "respect" the C++ precedence of operators. (What does that mean?) Give an unambiguous grammar G_2 for L which respects C++ precedence rules. Use three grammar variables: E for "expression," T for "term," and F for "factor." I will help get you started:

    1. $E \rightarrow E + T$
    2. $E \rightarrow E - T$
    3. $E \rightarrow T$
    4. $T \rightarrow T * F$
    5. $T \rightarrow$ (Missing rhs.)
    6. $\rightarrow -F$ (Missing lhs.)
    7. $F \rightarrow ($ $)$ (What goes inside the parentheses?)
    8. $F \rightarrow x$
    9. $F \rightarrow y$
   10. $F \rightarrow z$

4. Fill in the blanks.

   (a) If M_1 is an NFA with n states, then M_1 is equivalent to a DFA M_2 with no more than _____ states.

   (b) The _____ algorithm determines whether a given string of length n is generated by a given context-free grammar in $O(n^3)$ time.

   (c) A derivation tree of a string w generated by context-free grammar G can be labeled by writing, next to each variable symbol in the tree, the number of a production of G. The encoding of a left-most derivation of w is obtained by visiting the internal nodes of the corresponding derivation tree in _____. (The answer is either preorder, postorder, inorder, or level order.)

5. State the pumping lemma for regular languages by filling in the blanks.

If L is a _____ language, there exists an integer p, which we call the _____ of L, such that for every _____ such that _____ , there exist strings ____ , ____ and ____ such that the following four statements hold:

   1. _____
   2. _____
   3. _____
   4. For any _____ , the string _____ is a member of L.

6. Design a minimal DFA equivalent to the following NFA.
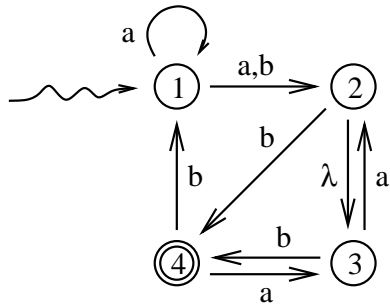
7. Design a PDA which accepts the language L of all even length palindromes over $\{a, b\}$,

$S \to aSa$
$S \to bSb$ which is generated by this context-free grammar. You may describe
$S \to \lambda$ your machine either by writing the transitions or by drawing a figure.

8. Let L be the language consisting of all binary strings with equally many zeros and ones. For example, 0100011011 and 1001 are in L, but 01101001100101101 is not. Design a DPDA which accepts L. You may describe your machine either by writing the transitions or by drawing a figure.

9. Let $L = \{a^n b^n c^n : n \geq 0\}$. As you know, L is not context-free.

    (a) Give two context-free languages whose intersection is L.

    (b) Give a context-sensitive grammar for L.

    (c) Give a context-free grammar for the complement of L. (This is hard. You need several variables.)

10. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

(i) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

(ii) _____ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

(iii) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

(iv) _____ The language $\{a^i b^j c^k \mid j = i + k\}$ is context-free.

(v) _____ The intersection of any three regular languages is regular.

(vi) _____ The intersection of any regular language with any context-free language is context-free.

(vii) _____ The intersection of any two context-free languages is context-free.

(viii) _____ The complement of any context-free language is context-free.

(ix) _____ If L is a context-free language over an alphabet with just one symbol, then L is regular.

(x) _____ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

(xi) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

(xii) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.

(xiii) _____ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

(xiv) _____ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

(xv) _____ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is decidable.

(xvi) _____

(xvii) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$-TIME.

(xviii) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

(xix) _____

(xx) _____ Every problem that can be mathematically defined has an algorithmic solution.

(xxi) _____

(xxii) _____ Every $\mathcal{NP}$ language is decidable.

(xxiii) _____ The intersection of two $\mathcal{NP}$ languages must be $\mathcal{NP}$.

(xxiv) _____ $\mathcal{P} = \mathcal{NP}$.

(xxv) _____ $\mathcal{NP} = \mathcal{P}$-SPACE

(xxvi) _____ Primality, where the input is in binary, is $\mathcal{P}$-TIME.

(xxvii) _____ Every context-free language is in $\mathcal{P}$.

(xxviii) _____ The problem of whether two given context-free grammars generate the same language is decidable.

(xxix) _____ If G is a context-free grammar, the question of whether $L(G) = \Sigma^*$ is decidable, where Σ is the terminal alphabet of G.

(xxx) _____ The language of all fractions (using base 10 numeration) whose values are less than π is decidable. (A *fraction* is a string. "314/100" is in the language, but "22/7" is not.)

(xxxi) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary ("caveman") numeral.

(xxxii) _____ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(xxxiii) _____ There exists a mathematical proposition that is true but cannot be proved.

(xxxiv) _____ There is an uncomputable function which grows faster than any computable function.

(xxxv) _____ There exists a machine that runs forever and outputs the decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(xxxvi) _____ For every real number x, there exists a machine that runs forever and outputs the decimal digits of x.

(xxxvii) _____ There is a polynomial time algorithm which determines whether any two regular expressions are equivalent.