University of Nevada, Las Vegas
Computer Science 456/656 Fall 2020

Practice for Final Examination December 8, 2021

The entire practice examination is 475 points. The real examination will be shorter.

1. True/False/Open See `tfI.pdf` and `tfII.pdf` for more T/F questions.

   (i) ________ Every subset of a regular language is regular.

   (ii) ________ Every context-free language is $\mathcal{NC}$.

   (iii) ________ $\mathcal{P}$–SPACE = EXP–SPACE.

   (iv) ________ Given a regular expression, an equivalent minimal DFA can always be constructed in polynomial time.

   (v) ________ $L = \{0^n1^n0^n1^n : n \geq 1\}$ is context-sensitive.

   (vi) ________ The intersection of two context-free lanuages must be context-free.

   (vii) ________ The concatenation of two context-free languages must be context-free.

   (viii) ________ The concatenation of two context-sensitive languages is context-sensitive.

   (ix) ________ The intersection of two co-RE languages is co-RE.

   (x) ________ Suppose a language L has matching delimeters. That is, its alphabet contains symbols ℓ and r, such that, in each $w \in L$, any instance of ℓ must be uniquely matched with an instance or r to its right. Then it is impossible for L to be regular.

   (xi) ________ The complement of every recursive language is recursive.

   (xii) ________ The complement of every recursively enumerable language is recursively enumerable.

   (xiii) ________ Every language which is generated by a general grammar is recursively enumerable.

   (xiv) ________ The set of all binary numerals for prime numbers is in the class $\mathcal{P}$.

   (xv) ________ If L_1 reduces to L_2 in polynomial time, and if L_1 is $\mathcal{NP}$, and if L_2 is $\mathcal{NP}$-complete, then L_1 must be $\mathcal{NP}$-complete.

   (xvi) ________ The union of any two context-free languages is context-free.

   (xvii) ________ The class of languages accepted by non-deterministic Turing machines is the same as the class of languages accepted by deterministic Turing machines.

   (xviii) ________ The class of languages accepted by non-deterministic push-down automata is the same as the class of languages accepted by deterministic push-down automata.

   (xix) ________ The intersection of any two context-free languages is context-free.

   (xx) ________ If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, then L_1 must be $\mathcal{NP}$.

   (xxi) ________ The language of all regular expressions over the binary alphabet is a regular language.

(xxii) ________ Let $e = \sum_{i=0}^{\infty} \frac{1}{i!} = 2.71828\ldots$, the base of the natural logarithm. The problem of whether the n^{th} digit of e, for a given n, is equal to a given digit is decidable.

(xxiii) ________ Every regular language is in the class $\mathcal{NC}$

(xxiv) ________ (Recall that $\langle x \rangle$ is the binary numeral for an integer x.) Let $x_1, x_2, \ldots$ be an arithmetic sequence of integers. The language $\{\langle x_i \rangle\}$ is regular.

(xxv) ________ (Recall that $\langle x \rangle$ is the binary numeral for an integer x.) Let $y_1, y_2, \ldots$ be a geometric sequence of integers. The language $\{\langle y_i \rangle\}$ is regular.

(xxvi) ________ The language of all binary strings which are the binary numerals for prime numbers is in the class $\mathcal{P}$–TIME.

(xxvii) ________ Every context-free grammar can be parsed by some non-deterministic top-down parser.

(xxviii) ________ If anyone ever proves that the integer factorization problem is $\mathcal{P}$–TIME, all public key/private key encryption systems will be known to be insecure.

(xxix) ________ If a string w is generated by a context-free grammer G, then w has a unique leftmost derivation if and only if it has a unique rightmost derivation.

(xxx) ________ The Boolean Circuit Problem is $\mathcal{NC}$.

(xxxi) ________ If there is an $\mathcal{NC}$ reduction from L_1 to L_2, and if L_2 is in Nick's class, then L_1 must be in Nick's class.

2. Every language, or problem, falls into exactly one of these categories. For each of the languages, write a letter indicating the correct category. [5 points each]

A Known to be $\mathcal{NC}$.
B Known to be $\mathcal{P}$–TIME, but not known to be $\mathcal{NC}$.
C Known to be $\mathcal{NP}$, but not known to be $\mathcal{P}$–TIME and not known to be $\mathcal{NP}$–complete.
D Known to be $\mathcal{NP}$–complete.
E Known to be $\mathcal{P}$–SPACE but not known to be $\mathcal{NP}$
F Known to be EXP–TIME but not nown to be $\mathcal{P}$–SPACE.
G Known to be EXP–SPACE but not nown to be EXP–TIME.
H Known to be decidable, but not nown to be EXP–SPACE.
K $\mathcal{RE}$ but not decidable.
L co-$\mathcal{RE}$ but not decidable.
M Neither $\mathcal{RE}$ nor co-$\mathcal{RE}$.

(a) ________ 3-CNF-SAT (usually known simply as 3-SAT)

(b) ________ 2-CNF-SAT (usually known simply as 2-SAT)

(c) ________ Halting problem.

(d) ________ Boolean circuit problem.

(e) ________ Regular grammar equivalence.

(f) ________ Context-free grammar equivalence.

(g) ________ Regular expression equivalence.

(h) ________ General sliding block problem.

(i) ________ Generalized checkers (any size rectangular board).

(j) ________ DFA equivalence.

(k) ________ All fractions whose values are less than $\sqrt{6}$.

(l) ________ $\{a^n b^n c^n d^n : !n \geq 1\}$

(m) ________ You have a number of containers, each of a given size and shape. You also have a set of objects of various sizes and shapes. Can you fit all the objects into the containers?

(n) ________ Dynamic programming, where each subproblem works in constant time and has Boolean output.

3. [20 points] Find a minimal DFA equivalent to the NFA shown in Figure 1.

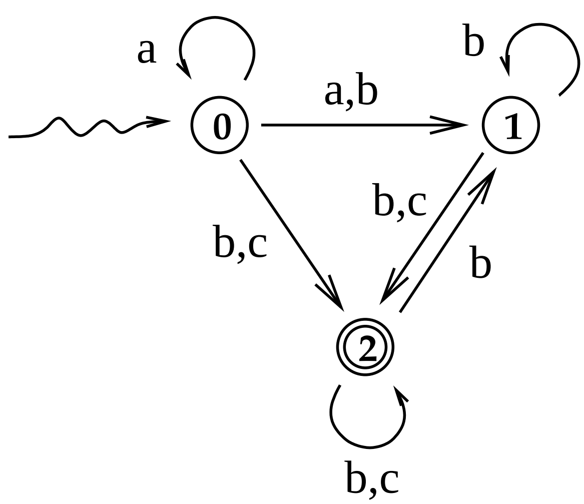

Figure 1: NFA for problem 3.

4. [20 points] Give a regular expression for the language accepted by the machine in Figure 2

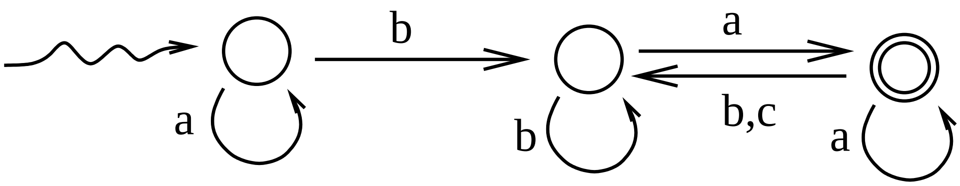

Figure 2: DFA for problem 4.

5. Which class of languages does each of these machine classes accept? [5 points each]

   (a) Deterministic finite automata. ______________________________

   (b) Non-deterministic finite automata. ______________________________

   (c) Push-down automata. ______________________________

   (d) Turing Machines. ______________________________

6. [20 points] The output of an LALR parser corresponds to the (pick one)

   (a) preorder

   (b) postorder

   (c) reverse preorder

   (d) reverse postorder

   visitation of the internal nodes of the parse tree.

7. [20 points] Design a PDA that accepts the Dyck language, whose grammar is given in problem 8

8. [20 points] The grammar below is an unambiguous CF grammar for the Dyck language, and is parsed by the LALR parser whose ACTION and GOTO tables are shown here. Write a computation of the parser for the input string *aabb*.

   1. $S \to S_{1,3}\, a_2 S_3 b_4$
   2. $S \to \lambda$

	a	b	\$	S
0	$r2$		$r2$	1
1	$s2$		halt	
2	$r2$	$r2$		3
3	$s2$	$s4$		
4	$r1$	$r1$	$r1$	

I will definitely give one of the following four proofs on the final exam.

9. [20 points] Prove that a recursively enumerable language is accepted by some machine.
10. [20 points] Prove that any language accepted by a machine is recursively enumerable.
11. [20 points] Prove that any decidable language is enumerated in canonical order by some machine.
12. [20 points] Prove that any language which can be enumerated in canonical order by some machine is decidable.
13. [20 points] Prove that the halting problem is undecidable. (This question will definitely be on the final exam.)

I will definitely give one of the following two reductions on the final exam.

14. [20 points] Give a polynomial time reduction of 3-SAT to the independent set problem.
15. [20 points] Give a polynomial time reduction of the subset sum problem to the partition problem.