CSC 456/656 Fall 2021 Answers to First Examination September 22, 2021

The entire test is 320 points.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of languages in $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.
   Each of these problems can be worked using what has been instroduced in class, although some of the problems require thinking, not just memorizing.

   (i) **F** Every subset of a regular language is regular.

   (ii) **T** $\mathcal{P} = \text{co-}\mathcal{P}$

   (iii) **O** $\mathcal{NP} = \text{co-}\mathcal{NP}$

   (iv) **T** DFA equivalence is $\mathcal{P}$.

   (v) **O** NFA equivalence is $\mathcal{P}$.

   (vi) **O** Regular expression equivalence is $\mathcal{P}$.

   (vii) **F** Context-free grammar equivalence is decidable.

   (viii) **T** Regular expression equivalence is decidable.

   (ix) **T** The class of regular languages is closed under intersection.

   (x) **T** The class of regular languages is closed under Kleene closure.

   (xi) **T** The class of context-free languages is closed under union.

   (xii) **T** The class of context-free languages is closed under concatenation.

   (xiii) **F** The class of context-free languages is closed under intersection.

   (xiv) **F** The class of context-free languages is closed under complementation.

   (xv) **T** The class of context-free languages is closed under Kleene closure.

   (xvi) **F** The language L_{HALT} is decidable.

   (xvii) **T** Every context-free language is in $\mathcal{P}$.

   (xviii) **T** There exists a function from integers to integers which grows faster than any computable function from integers to integers.

(xix) **T** We define a set of integers to be *regular* if the set of unary numerals for those integers is a regular language. If S is the set of terms of an arithmetic sequence, (such as $\{21, 25, 29, \ldots\}$) then S is regular.

(xx) **F** The set of binary numerals for prime numbers is a regular language.

(xxi) **T** The complement of L is context-free, where $L = \{a^n b^n c^n : n \geq 0\}$.

(xxii) **T** If a language L is accepted by some PDA, then L must be generated by some context-free grammar.

(xxiii) **F** Two PDAs are said to be *equivalent* if they accept the same language. Every PDA is equivalent to some DPDA.

(xxiv) **F** If L has an unambiguous CF grammar, than there must be a DPDA which accepts L.

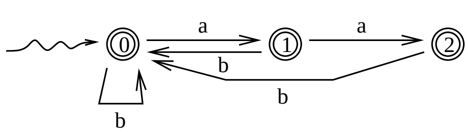
(xxv) **F** LALR parsers are used to parse context-free languages. In order to use such a parser, it is necessary that the CF grammar be unambigous.

(xxvi) **O** $\mathcal{P} = \mathcal{NP}$.

2. [10 points] Name a problem which is known to be $\mathcal{NP}$ and is also known to be co-$\mathcal{NP}$, but is not known to be $\mathcal{P}$.

   Factoring binary numerals.

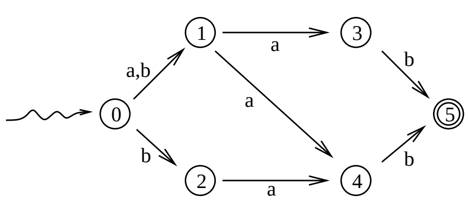
3. [20 points] Draw a DFA which accepts the language over $\{a, b\}$ consting of all strings which do not have aaa as a substring.



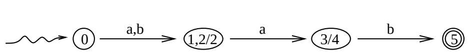
4. [20 points] let L be the language consisting of all strings w over $\{a, b\}$ such that the third-from-last symbol of w is a. Write a regular expression for L.

   $(a + b)^* a(a + b)(a + b)$

5. [20 points] Consider the NFA M pictured below.



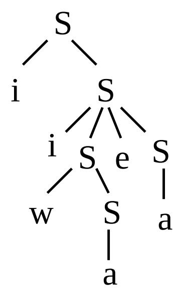
   Construct a minimal DFA equivalent to M.

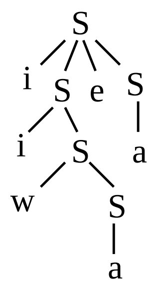


6. [20 points] Let G be the CF grammar given below.

1. $S \to a$
2. $S \to wS$
3. $S \to iS$
4. $S \to iSeS$

Show that G is ambiguous by drawing two different derivation trees for the string $iiwaea$.



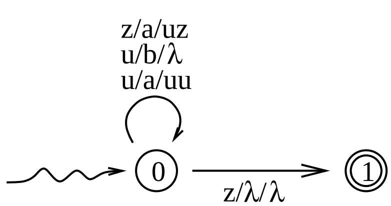


This grammar is a tiny model of a programming language, where
1. S means statement
2. a means assignment statement
3. w means while
4. i means if
5. e means else

7. [20 points] Give a context-free grammar for $L = \left\{ a^i b^j c^k \,:\, i = j + k \right\}$

$S \to aSc$
$S \to T$
$T \to aTb$
$T \to \lambda$

8. [20 points] Design a DPDA which accepts the Dyck language.



9. [20 points] State the pumping lemma for regular languages.

For any regular language L there is an integer p, called the pumping length of L, such that for any $w \in L$ of length at least p, there are strings x, y, and z such that the following four conditions hold:
1. $w = xyz$
2. $|xy| \leq p$
3. y is not the empty string
4. For any integer $i > 0$, $xy^iz \in L$

10. [20 points] Prove that $\sqrt{2}$ is irrational. By contradiction. Assume $\sqrt{2}$ is rational. Then $\sqrt{2}$ is equal to a fraction reduced to the lowest terms, that is, $\sqrt{2} = \frac{p}{q}$ where p, q are integers which have no common divisor greater than 1.

Squaring both sides, we have
$2 = \frac{p^2}{q^2}$
$2q^2 = p^2$
Thus p^2 is even, which implies that p is even.
We write $p = 2k$ for some integer k
$p^2 = 4k^2$
$2q^2 = p^2 = 4k^2$
$q^2 = 2k^2$
q^2 is even, which implies that q is even.
Thus 2 is a common divisor of p, q, constradiction. We conclude that $\sqrt{2}$ is irrational.

11. [20 points] Give a context-sensitive grammar for $L = \{a^n b^n c^n \, : \, n \geq 1\}$.

I have given an answer, and there is another in the textbook. Here is a very simple answer. (But is it correct?)
1. $S \to abc$
2. $ab \to aabbA$
3. $Ab \to bA$
4. $Ac \to cc$

Here is a derivation of $aaabbbccc$. $u \Rightarrow v$, where u and v are sentential forms, the number of the production is written over the double arrow. The left hand side of that production is the underlined substring of u, which is then replaced by the right hand side of that production to form v.

$\underline{S} \stackrel{1}{\Rightarrow} \underline{ab}c \stackrel{2}{\Rightarrow} aabb\underline{Ac} \stackrel{4}{\Rightarrow} a\underline{ab}bcc \stackrel{2}{\Rightarrow} aaabb\underline{Ab}cc \stackrel{3}{\Rightarrow} aaabbb\underline{Ac}c \stackrel{4}{\Rightarrow} aaabbbccc$