

Assignment 5: Due Friday October 21 2022, 11:59 PM

Name: \_\_\_\_\_

You are permitted to work in groups, get help from others, read books, and use the internet. Turn in the assignment in the manner given to you by our grader, Janeen Sudiacal.

1. True, false, or open:

- (a) \_\_\_\_\_ Every context-free language is in the class  $\mathcal{P}$ -TIME.
- (b) \_\_\_\_\_ There is an  $\mathcal{NP}$ -complete problem which is not  $\mathcal{P}$ -TIME.
- (c) \_\_\_\_\_ The problem 3-SAT is known to be  $\mathcal{NP}$ -complete.
- (d) \_\_\_\_\_ The problem 2-SAT is known to be  $\mathcal{NP}$ -complete.
- (e) \_\_\_\_\_ Every context-sensitive language is decidable.

2. Most new  $\mathcal{NP}$ -complete problems are found by giving a polynomial time reduction from a problem which is already known to be  $\mathcal{NP}$ -complete.

The Roughgarden lectures are on Youtube. The lecture at

[https://www.youtube.com/watch?v=sbi7kspZPZs&list=RDCMUCcH4Ga14Y4ELFKrEYM1vXCg&start\\_radio=1&rv=sbi7kspZPZs&t=340](https://www.youtube.com/watch?v=sbi7kspZPZs&list=RDCMUCcH4Ga14Y4ELFKrEYM1vXCg&start_radio=1&rv=sbi7kspZPZs&t=340)

gives a polynomial time reduction of the Independent Set problem to the Subset Sum problem. Existence of this reduction proves that the Subset Sum problem is  $\mathcal{NP}$ -complete, given that the Independent Set problem is  $\mathcal{NP}$ -complete.

3. Give a polynomial time reduction of the Subset Sum problem to the Partition problem, sometimes called the Set-partition problem.

4. Correctly state (do not prove) the pumping lemma for context-free languages.

5. Let  $L$  be the set of all strings of the form  $\langle G_1 \rangle \langle G_2 \rangle$  such that  $G_1$  and  $G_2$  are context-free grammars that are not equivalent.  $L$  is not context-free, since if it were, the CF grammar equivalence problem would be decidable. Prove that  $L$  is RE. You may assume that the description of a CF grammar uses the alphabet  $\Sigma$ .

6. Consider the CF grammar  $G$  given below. The strings are algebraic expressions where there is only one operator, subtraction, and only one variable,  $x$ , and where parentheses can be used in the usual way. Fill in the ACTION and GOTO tables for an LALR parser that parses this grammar.

To make things easier for you:

- (a) I have included stack states as subscripts in the definition of the grammar.
- (b) Some of the entries are already written.
- (c) The ACTION and GOTO tables have many places where there should be no entry. I have placed an X in each of those places.

Note that  $G$  is ambiguous. Can you identify the shortest string which is generated by  $G$  in two different ways?

There is exactly one entry in the ACTION table which resolves this ambiguity, so that the output derivation is consistent with the usual semantics of algebraic expressions. Which entry is this?

- 1.  $E \rightarrow E -_2 E_3$
- 2.  $E \rightarrow ({}_4 E_5)_6$
- 3.  $E \rightarrow x_7$

	ACTION					GOTO
	$x$	$-$	$($	$)$	$\$$	$E$
0	$s7$	X	$s4$	X	X	1
1	X	$s2$	X	X	HALT	X
2		X		X	X	
3	X	$r1$	X			X
4		X		X	X	
5	X		X		X	X
6	X	$r2$	X			X
7	X	$r3$	X			X