University of Nevada, Las Vegas Computer Science 456/656 Spring 2022

Assignment 7: Due Friday December 9 2022, 11:59 PM

This version: Sat Dec 10 21:37:02 PST 2022

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet. Turn in the assignment in the manner given to you by our grader, Janeen Sudiacal.

I have given answers to some of the problems, including all the true false problems; thus, the homework is shorter than it looks. But be sure to go over those answers.

If you notice any errors, email me **immediately.**

1. True or False. If the question is currently open, write "O" or "Open."

   (i) **T** The complement of every regular language is regular.

   (ii) **F** The complement of every context-free language is context-free.

   (iii) **T** The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

   (iv) **O** The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

   (v) **T** The complement of any $\mathcal{P}$-SPACE language is $\mathcal{P}$-SPACE.

   (vi) **T** The complement of every recursive language is recursive.

   (vii) **F** The complement of every recursively enumerable language is recursively enumerable.

   (viii) **T** Every language which is generated by a general grammar is recursively enumerable.

   (ix) **F** The context-free membership problem is undecidable.

   (x) **T** The factoring problem, where inputs are written in binary notation, is co-$\mathcal{NP}$.

   (xi) **T** If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, and if L_1 is $\mathcal{NP}$-complete, then L_2 must be $\mathcal{NP}$-complete.

   (xii) **F** Given any context-free grammar G and any string $w \in L(G)$, there is always a unique leftmost derivation of w using G.

   (xiii) **F** For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

   (xiv) **F** The question of whether two regular expressions are equivalent is known to be $\mathcal{NP}$-complete.

   (xv) **T** The halting problem is recursively enumerable.

   (xvi) **T** The union of any two context-free languages is context-free.

   (xvii) **F** The question of whether a given Turing Machine halts with empty input is decidable.

(xviii) **T** The class of languages accepted by non-deterministic finite automata is the same as the class of languages accepted by deterministic finite automata.

(xix) **F** The class of languages accepted by non-deterministic push-down automata is the same as the class of languages accepted by deterministic push-down automata.

(xx) **T** The class of languages accepted by non-deterministic Turing Machines is the same as the class of languages accepted by deterministic Turing Machines.

(xxi) **F** The intersection of any two context-free languages is context-free.

(xxii) **T** If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, then L_1 must be $\mathcal{NP}$.

(xxiii) **T** Let π be the ratio of the circumference of a circle to its diameter. The problem of whether the n^{th} digit of the decimal expansion of π for a given n is equal to a given digit is decidable.

(xxiv) **T** There cannot exist any computer program that can decide whether any two C++ programs are equivalent.

(xxv) **T** Every context-free language is in the class $\mathcal{P}$-TIME.

(xxvi) **T** Every regular language is in the class $\mathcal{NC}$

(xxvii) **T** The language of all binary numerals for multiples of 23 is regular.

(xxviii) **F** The language of all binary strings which are the binary numerals for prime numbers is context-free.

(xxix) **T** Every context-free grammar can be parsed by some non-deterministic top-down parser.

(xxx) **T** If anyone ever proves that $\mathcal{P} = \mathcal{NP}$, then all one-way encoding systems will be insecure.

(xxxi) **T** If a string w is generated by a context-free grammer G, then w has a unique leftmost derivation if and only if it has a unique rightmost derivation.

(xxxii) **T** A language L is in $\mathcal{NP}$ if and only if there is a polynomial time reduction of L to SAT.

(xxxiii) **F** Every subset of a regular language is regular.

(xxxiv) **T** The intersection of any context-free language with any regular language is context-free.

(xxxv) **T** Every language which is generated by a general grammar is recursively enumerable.

(xxxvi) **T** The question of whether two context-free grammars generate the same language is undecidable.

(xxxvii) **T** There exists some proposition which is true but which has no proof.

(xxxviii) **T** The set of all binary numerals for prime numbers is in the class $\mathcal{P}$.

(xxxix) **T** If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, and if L_1 is $\mathcal{NP}$-complete, then L_2 must be $\mathcal{NP}$-complete.

(xl) **F** Given any context-free grammar G and any string $w \in L(G)$, there is always a unique leftmost derivation of w using G.

(xli) **F** For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

(xlii) **O** The question of whether two regular expressions are equivalent is $\mathcal{NP}$-complete.

(xliii) **F** No language which has an ambiguous context-free grammar can be accepted by a DPDA.

(xliv) **T** The union of any two context-free languages is context-free.

(xlv) **F** The question of whether a given Turing Machine halts with empty input is decidable.

(xlvi) **T** The class of languages accepted by non-deterministic finite automata is the same as the class of languages accepted by deterministic finite automata.

(xlvii) **F** The class of languages accepted by non-deterministic push-down automata is the same as the class of languages accepted by deterministic push-down automata.

(xlviii) **T** The intersection of any two regular languages is regular.

(xlix) **F** The intersection of any two context-free languages is context-free.

(l) **T** If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, then L_1 must be $\mathcal{NP}$.

(li) **T** Let $F(0) = 1$, and let $F(n) = 2^{F(n-1)}$ for $n > 0$. Then F is recursive.

(lii) **T** Every language which is accepted by some non-deterministic machine is accepted by some deterministic machine.

(liii) **F** The language of all regular expressions over the binary alphabet is a regular language.

(liv) **T** Let π be the ratio of the circumference of a circle to its diameter. (That's the usual meaning of π you learned in school.) The problem of whether the n^{th} digit of π, for a given n, is equal to a given digit is decidable.

(lv) **T** There cannot exist any computer program that decides whether any two given C++ programs are equivalent.

(lvi) **F** An undecidable language is necessarily $\mathcal{NP}$-complete.

(lvii) **T** Every context-free language is in the class $\mathcal{P}$-TIME.

(lviii) **T** Every regular language is in the class $\mathcal{NC}$

(lix) **F** Every function that can be mathematically defined is recursive.

(lx) **T** The language of all binary strings which are the binary numerals for multiples of 23 is regular.

(lxi) **F** The language of all binary strings which are the binary numerals for prime numbers is context-free.

(lxii) **F** Every bounded function from integers to integers is Turing-computable. (We say that f is *bounded* if there is some B such that $|f(n)| \leq B$ for all n.)

(lxiii) **F** The language of all palindromes over $\{0, 1\}$ is inherently ambiguous.

(lxiv) **F** Every context-free grammar can be parsed by some deterministic top-down parser.

(lxv) **T** Every context-free grammar can be parsed by some non-deterministic top-down parser.

(lxvi) **F** Commercially available parsers cannot use the LALR technique, since most modern programming languages are not context-free.

(lxvii) **F** The boolean satisfiability problem is undecidable.

(lxviii) **T** If anyone ever proves that $\mathcal{P} = \mathcal{NP}$, then all public key/private key encryption systems will be known to be insecure.

(lxix) **T** If a string w is generated by a context-free grammer G, then w has a unique leftmost derivation if and only if it has a unique rightmost derivation.

(lxx) **F** If a machine outputs a sequence of fractions which converges to a real number x, then x must be a recursive real number.

(But, do you see why?)

2. What class of machines accepts the class of context free languages?

3. What class of machines accepts the class of recursively enumerable languages?

4. Using the context-free grammar with start symbol S and productions listed below, write two different leftmost derivations (not parse trees) of the string *iibwaanea*

$S \to a$
$S \to bLn$
$S \to wS$
$S \to iS$
$S \to iSeS$
$L \to \lambda$
$L \to SL$

5. Draw an NFA with five states which accepts the language described by the regular expression

$(a+b)^*a(a+b)(a+b)(a+b)$

6. Draw a DFA which accepts the language L over the alphabet $\{a, b, c\}$ consisting of all strings which contain either *aba* or *caa* as a substring. (My answer has six states.)

7. Find a context-free grammar which generates the language $L = \left\{a^i b^j c^k \ : \ i = j \text{ or } i = k\right\}$

8. Draw a state diagram for a PDA that accepts the Dyck language. (For ease of grading, use a and b instead of "(" and ")")

9. Every language we have discussed this semester falls into at least one of these categories.

   a. $\mathcal{NC}$.
   b. $\mathcal{P}$ but not known to be $\mathcal{NC}$.
   c. $\mathcal{NP}$ but not known to be $\mathcal{P}$ and not known to be $\mathcal{NP}$-complete.
   d. Co-$\mathcal{NP}$ but not known to be $\mathcal{P}$.
   e. Known to be $\mathcal{NP}$-complete.
   f. Recursive, but not known to be $\mathcal{NP}$.
   g. RE (Recursively enumerable), but not recursive.
   h. Co-RE, but not recursive.
   i. Neither RE nor co-RE.

   State which of the above categories each of the languages below falls into.

   (ii) _____ The 0-1 Traveling Salesman Problem.

   (iii) _____ The diagonal language.

   (iv) _____ L_{sat}, the set of satisfiable boolean expressions.

   (v) _____ The language described by the regular expression a^*b*.

   (vi) _____ The Boolean circuit problem.

   (vii) _____ Dynamic programming where all each subproblem can be worked in polynomial time and has Boolean output.

   (viii) _____ The language $\{a^n b^n \ : \ n \geq 0\}$.

(ix) _____ Factorization of a binary numeral.

(x) _____ Dynamic programming where all subproblems have Boolean solutions, and each subproblem can be computed using only the output of the immediately previous subproblem.

(xi) _____ Multiplication of two binary numerals.

(xii) _____ The 0-1 traveling salesman problem.

(xiii) _____ The set of configurations of Rush Hour which are solvable.

(xiv) _____ Factorization of a unary numeral.

(xv) _____ The halting problem.

(xvi) _____ The clique problem.

(xvii) _____ Primality, where the input is written in binary.

(xviii) _____ The language generated by a given context-free grammar.

(xix) _____ The language of all finite monotone increasing sequences of arabic numerals for positive integers. (For example, "1,5,23,41,200,201" is a member of that language.)

(xx) _____ The language accepted by a given DFA.

(xxi) _____ The context-free grammar equivalence problem.

(xxii) _____ The independent set problem.

(xxiii) _____ The language consisting of all C++ programs P such that P does not halt if given P as its input file.

(xxiv) _____ 3-CNF-SAT.

(xxv) _____ The language described by the regular expression $\texttt{a(b+bc)}^*\texttt{(}\lambda\texttt{+a)(ba)}^*$

10. Draw the state diagram for a DFA that accepts the language described by the regular expression $\texttt{(a(}\lambda\texttt{+b+bb)a)}^*$

11. Let L be the language of all binary numerals for positive integers which are multiples of 4. Thus, for example, the binary numerals for 0, 4, 8, 12, 16, 20 ... are in L. We allow a binary numeral to have leading zeros; thus (for example) $0011100 \in L$, since it is a binary numeral for 28. Draw a DFA with four states which accepts L.

12. What does it mean to say that machines M_1 and M_2 are *equivalent*?

13. Prove that every decidable language can be enumerated in canonical order by some machine.

14. State the Church-Turing Thesis.

15. Give a $\mathcal{P}$-time reduction of the subset sum problem to the partion problem.

16. Draw the state diagram for a minimal DFA that accepts the language described by the regular expression $\texttt{a}^*\texttt{a(b+ab)}^*$

17. Write a regular expression for the language accepted by the NFA shown in Figure 1.

18. Let L be the language of all binary numerals for positive integers equivalent to 2 modulo 3. Thus, for example, the binary numerals for 2, 5, 8, 11, 14, 17 ... are in L. We allow a binary numeral to have leading zeros; thus (for example) $001110 \in L$, since it is a binary numeral for 14. Draw a minimal DFA which accepts L.

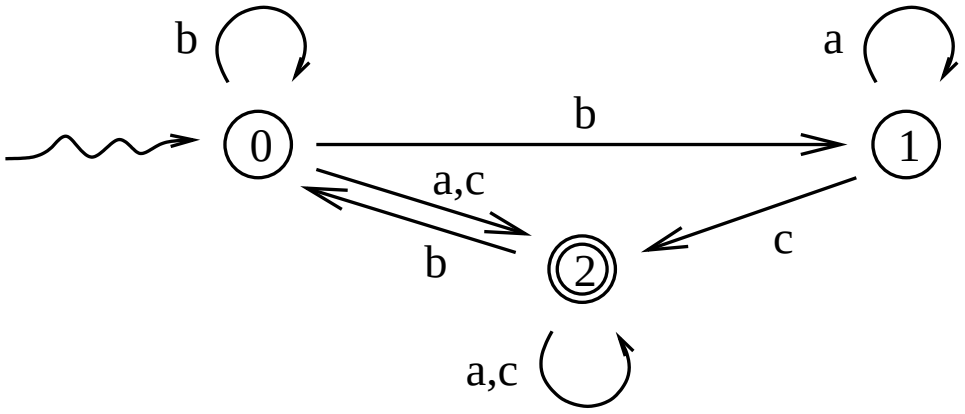


Figure 1: The NFA for Problems 17 and 32.

19. Design a PDA that accepts the language of all palindromes over the alphabet $\{a, b\}$.

20. Consider the context-free grammar with start symbol S and productions as follows:

    $S \to s$
    $S \to bLn$
    $S \to wS$
    $L \to \lambda$
    $L \to SL$

    Write a leftmost derivation of the string *bswsbwsnn*

21. What class of machines accepts the class of context free languages?

22. What class of machines accepts the class of regular languages?

23. What is the Church-Turing Thesis, and why is it important?

    Every computation that can be done by any machine can be done by a Turing machine. This is important because, if we can prove that no Turing machine can perform a certain computation, we know that no machine can perform that computation. Turing machines are simple, making these proofs easier.

24. What does it mean to say that a language can be recursively enumerated in *canonical order*? What is the class of languages that can be so enumerated?

    If u and v are strings, we say that u comes before v in the canonical the canonical order if either $|u| < |v|$, or $|u| = |v|$ and u comes before v in lexical [alphabetic] order.

We say that a language L can be enumerated in canonical order if there is some machine with no input which outputs all the strings of L in canonical order, and does not output any other strings.

A language L can be enumerated in canonical order if and only if it is recursive [decidable].

25. What does it mean to say that machines M_1 and M_2 are *equivalent*?

26. Give a definition of the language class $\mathcal{NP}$–TIME. Two definitions were given in class.

27. Give the definition of a *polynomial time reduction* of a language L_1 to another language L_2.

    Let Σ_1, σ_2 be the alphabets of L_1 and L_2, respectively. A polynomial time reduction of L_1 to L_2 is a function $R : \Sigma_1^* \to \Sigma_2^*$ which is computable in polynomial time such that, for any $w \in \Sigma_1^*$, $w \in L_1$ if and only if $R(w) \in L_2$.

28. Give a definition of $\mathcal{NP}$-*complete* language. A language L is $\mathcal{NP}$-complete if

    (a) L is in the class $\mathcal{NP}$–TIME

    (b) For any language L_2 in the class $\mathcal{NP}$-TIME, there is a polynomial time reduction of L_2 to L.

29. Give a definition of a *decidable* language.

    We say that $L \subseteq \Sigma^*$ is decidable if there is a machine M such that, given $w \in \Sigma^*$, M halts with input w, and outputs 1 if $w \in L$ and outputs 0 if $w \notin L$.

30. We say a binary string w over is *balanced* if w has the same number of 1's as 0's. Let L be the set of balanced binary strings. Give a context-free grammar for L.

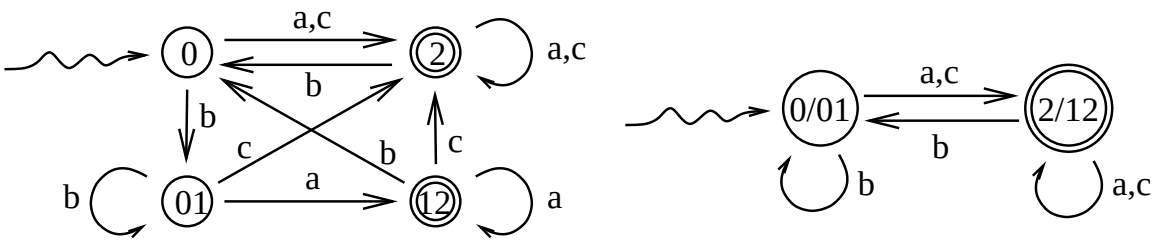
    Here is a simple ambiguous context-free grammar for L.
    $S \to SS$
    $S \to aSb$
    $S \to bSa$
    $S \to \lambda$

    Here is an unambiguous grammar for L.
    $S \to aAbS$
    $S \to bBaS$
    $S \to \lambda$
    $A \to aAbA$

$A \to \lambda$
$B \to bBaB$
$B \to \lambda$

31. Give a Chomsky normal form (CNF) grammar for the language of all palindromes of odd length over the alphabet $\{a, b\}$.

32. Construct a minimal DFA equivalent equivalent to the NFA shown in Figure 1.



The first figure is the DFA obtained by the powerset method. The second figure is obtained by minimization.

33. Consider the context-free grammar G, with start symbol S and productions as follows:
$S \to s$
$S \to bLn$
$S \to iS$
$S \to iSeS$
$L \to \epsilon$
$L \to LS$
Prove that G is ambiguous by giving two different leftmost derivations for some string.
$S \Rightarrow iSeS \Rightarrow iiSeS \Rightarrow iiaeS \Rightarrow iiaea$
$S \Rightarrow iS \Rightarrow iiSeS \Rightarrow iiaeS \Rightarrow iiaea$

34. The grammar below is an alternative unambiguous CF grammar for the Dyck language, and is parsed by the given LALR parser. Write a computation of the parser for the input string *aabb*.

1 $S \to a_2 S_3 b_4 S_5$

2 $S \to \lambda$

	a	b	\$	S
0	$s2$		$r2$	1
1			HALT	
2	$s2$	$r2$		3
3		$s4$		
4	$s2$	$r2$	$r2$	5
5		$r1$	$r1$	

35. The following ambiguous grammar, with start symbol E, generates certain expressions. The parser illustrated below is intended to parse a string in accordance with C++ rules. However, the action table has one error. Identify that error, and fix it.

$E \to x_2$

$E \to E - {}_3E_4$

$E \to -_5 E_6$

$E \to ({}_7E_8)_9$

	x	$-$	(	)	\$	S
0	$s2$	$s5$	$s7$			1
1		$s3$			HALT	
2		$r1$		$r1$	$r1$	
3	$s2$	$s5$	$s7$			4
4		$r2$		$r2$	$r2$	
5	$s2$	$s5$	$s7$			6
6		$r3$		$r3$	$r3$	
7	$s2$	$s5$	$s7$			8
8		$s3$		$s9$		
9		$r4$		$r4$	$r4$	

36. For each of the following languages, state whether the language is regular, context-free but not regular, context-sensitive but not context-free, or not context-sensitive.

(a) **Regular.** The set of all strings over the alphabet $\{a, b\}$ of the form $a^n b^m$.

(b) **Context-free, not regular.** The set of all strings over the alphabet $\{a, b\}$ of the form $a^n b^n$.

(c) **Context-sensitive, not context-free.** The set of all strings over the alphabet $\{a, b, c\}$ of the form $a^n b^n c^n$.

(d) **Context-free, not regular.** The set of all strings over the alphabet $\{a, b, c\}$ which are **not** of the form $a^n b^n c^n$.

37. Draw a minimal DFA which accepts the language L over the binary alphabet $\Sigma = \{a, b, c\}$ consisting of all strings which contain either *aba* or *caa* as a substring.

See problem 6.

38. State the pumping lemma for regular languages accurately. If you have all the right words but in the wrong order, that means you truly do not understand the lemma, and you might get no partial credit at all.

39. These are reduction problems. I could give one of them on the test. The proof should be very informal.

    (a) Find a $\mathcal{P}$-time reduction of 3-CNF-SAT to the independent set problem.

    (b) Give a $\mathcal{P}$-time reduction of the subset sum problem to the partition problem.
    See Problem 15

40. Prove that every recursively enumerable language is accepted by some machine.

    Let L be a recursively enumerable language; let M be a machine that enumerates L. We let our machine be a program which reads a string w and halts if and only if $w \in L$. Let $w_1, w_2, \ldots$ be the strings of L in the order they are enumerated by M.

    Here is the program:

    Read w
    For $i = 1, 2, \ldots$
      If $(w = w_i)$ HALT

41. Prove that every language accepted by a machine is recursively enumerable.

    Let M be a Turing machine that accepts a language L over an alphabet Σ. Let $w_1, w_2, \ldots$ be an enumeration of Σ^* in canonical order. (That enumeration is easy to compute.) The following program runs forever, enumerating L.

For $(t = 1, 2, \ldots)$ // infinite loop
  For $(i = 1, 2, \ldots t)$ // finite loop
    If (M accepts w_i within t steps)
      WRITE w_i.

If w_i is written, then M clearly accepts w_i, hence $w_i \in L$. Conversely, if $w_i \in L$, then M accepts L within t steps for some t, and hence will be written during the t^{th} iteration of the outer loop.

42. Prove that the halting problem is undecidable.

43. Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

$S \to a$
$E \to a$
$S \to LA$
$E \to LA$
$L \to ($
$A \to ER$
$R \to)$

$S \to PE$
$E \to PE$
$S \to EE$
$E \to EE$
$P \to EQ$
$Q \to +$

Use the CYK algorithm to prove that the string $a(a + a)$ is a member of L. Use the figure below for your work.