University of Nevada, Las Vegas Computer Science 456/656 Spring 2021

Answers to Assignment 3: Due Friday September 23 2022, 11:59 PM

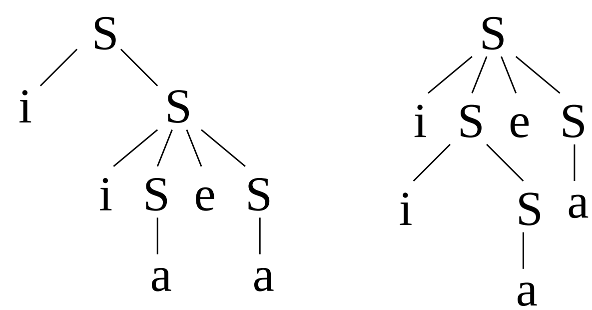
1. Correctly state (not prove) the pumping lemma for regular languages.

   It is important to have the nested quantifiers in the correct order. If not, you might have all the right words, but still get zero on this problem. I will emphasize the quantifiers.

   For any regular language L, there is an integer p such that for any string $w \in L$ of length at least p, there are strings x, y, z such that the following conditions hold:
   1. $w = xyz$
   2. $|xy| \le p$
   3. y is not the empty string
   4. for any $i \ge 0$, $xy^iz \in L$.

2. Let G be the following context-free grammar:
   1. $S \to a$
   2. $S \to iS$
   3. $S \to iSeS$

   Prove that G is ambiguous by giving two different parse trees of the string `iiaea`. Both of those parse trees are correct, but nly one of the two parse trees is consistent with the usual rule for resolving ambiguity of if-then-else statements. Which one?



   The tree on the left agrees with the usual programming rulse that an `else` matches with the nearest unmatched `if` to its left.

3. The Dyck language is the language of balanced strings of left and right parentheses. For example, the strings $(), (()), ()(()), \lambda$ are members of the Dyck language, but $)(, (()$ are not.

   Here is an unambiguous grammar for the Dyck language.

   1. $S \to S(S)$
   2. $S \to \lambda$

   Using that grammar, write a left-most derivation for the string $(())$, then write a right-most derivation for the same string.

   leftmost: $S \stackrel{1}{\Rightarrow} S(S) \stackrel{2}{\Rightarrow} (S) \stackrel{1}{\Rightarrow} (S()) \stackrel{2}{\Rightarrow} (())$

   rightmost: $S \stackrel{1}{\Rightarrow} S(S) \stackrel{1}{\Rightarrow} S(S()) \stackrel{2}{\Rightarrow} S(()) \stackrel{2}{\Rightarrow} (())$

   The production numbers over the double arrows are optional.

4. Give a context-free grammar for the language $\{a^nb^m : 0 \le m = 2n\}$
   1. $S \to aSbb$
   2. $S \to \lambda$

5. You know that the intersection of two CF languages may not be CF. Here is an example. Using the pumping lemma for context-free languages, it is possible to prove that $L = \{a^n b^n c^n \; : \; n \geq 0\}$ is not context-free.

(a) Give context-free grammars for the languages L_1 and L_2, where

$L_1 = \{a^n b^m c^m \; : \; n \geq 0, m \geq 0\}$

$L_2 = \{a^n b^n c^m \; : \; n \geq 0, m \geq 0\}$

G_1 generates L_1:
1. $S \rightarrow AB$
2. $A \rightarrow aA$
3. $A \rightarrow \lambda$
4. $B \rightarrow bBc$
5. $B \rightarrow \lambda$

G_2 generates L_2:
1. $S \rightarrow AB$
2. $A \rightarrow aAb$
3. $A \rightarrow \lambda$
4. $B \rightarrow cB$
5. $B \rightarrow \lambda$

(b) Explain why $L_1 \cap L_2$ is not context-free.

$L_1 \cap L_2 = L$ which is not context free.