Push Down Automata and Deterministic Push Down Automata

Definitions

We use the following definitions given in your textbook, Formal Languages and Automata, by Peter Linz.

Definition 1 *If Σ is any alphabet, let $\Sigma_\lambda = \Sigma \cup \{\lambda\}$. (Most books use ε instead of γ, but we'll stick to the notation in our textbook.)*

Definition 2 *If S is any set, $\mathcal{P}(S) = 2^S$ is the set of all subsets of S. We write $\mathcal{F}(S) \subseteq \mathcal{P}(S)$ to be the set of all* **finite** *subsets of S.*

Definition 3 *A PDA is a septuple (7-tuple) $M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$ where*

1. *Q is the set of states,*
2. *Σ is the input alphabet,*
3. *Γ is the stack alphabet,*
4. *$\delta : Q \times \Sigma_\delta \times \Gamma \longrightarrow \mathcal{F}(Q \times \Gamma^*)$ is the transition function,*
5. *$q_0 \in Q$ is the start state, and*
6. *$z \in \Gamma$ is the bottom-of-stack symbol.*
7. *$F \subseteq Q$ is the set of accept states.*

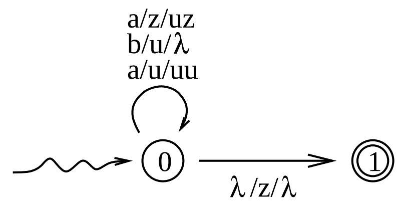
Configurations and Computations of a PDA

If $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ is a PDA, we define a *configuration* of M to be an ordered triple $\mathbf{i} = (\sigma, q, w)$ where $\sigma \in \Gamma^*$ is the current stack, $q \in Q$ is the current state, and $w \in \Sigma^*$ is the current (unread) input.

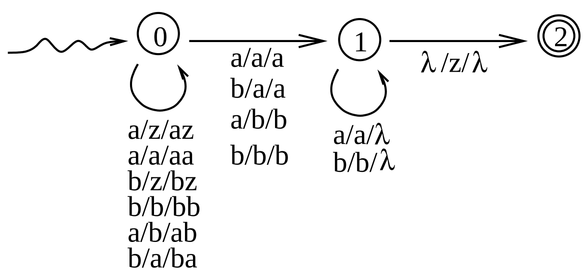
The *start configuration* of M is the configuration (z, q_0, w) for some $w \in \Sigma^*$. A configuration (σ, q, w) is *final* if $q \in F$ and $w = \lambda$. If $\mathbf{i}$ and $\mathbf{i}'$ are configurations of M, we write $\mathbf{i} \mapsto \mathbf{i}'$ to mean that, if M is in configuration $\mathbf{i}$, then M can possibly read a symbol, pop the top symbol off stack, then push any string onto the stack, then change its configuration to $\mathbf{i}'$. We say that M is *deterministic* if, for each $\mathbf{i}$, there is at most one choice of $\mathbf{i}'$.

A *computation* of M is a finite sequence of configurations $\mathbf{i}_0, \mathbf{i}_1 \ldots \mathbf{i}_n$ such that $\mathbf{i}_i \mapsto \mathbf{i}_{i+1}$ for all i $\mathbf{i}_0$ is a start configuration of M. and $\mathbf{i}_n$ is a final configuration of M.

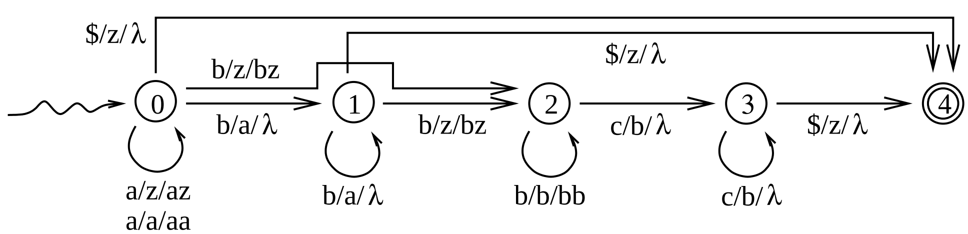
Examples



$\mathbf{a}^n b^n$



Palindromes



What is the language?