CSC 456/656 Fall 2022 Answers to First Examination September 28, 2022

The entire test is 230 points.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of members of $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

(i) **F** Every subset of a regular language is regular.
Every language is a subset of a regular language.

(ii) **F** Context-free grammar equivalence is decidable.
However, it is co-RE.

(iii) **T** Regular grammar equivalence is decidable.

(iv) **T** The class of regular languages is closed under intersection.

(v) **T** The class of regular languages is closed under Kleene closure.

(vi) **T** The class of context-free languages is closed under union.

(vii) **F** The class of context-free languages is closed under intersection.
$\{a^n b^n c^n \,:\, n \geq 0\}$ is the intersection of two context-free languages, but is not context-free.

(viii) **F** The set of binary numerals for prime numbers is a regular language.
Well, you should be aware that this is considered to be a very hard problem, although it was (finally) proved to be $\mathcal{P}$–TIME recently. If the language were regular, there would be a fast algorithm to decide primality. (This is not really a proof, just a "convince.")

(ix) **T** If a language L is accepted by some PDA, then L must be generated by some context-free grammar.
Well-known theorem. I haven't proved it in class, since it would take more than one lecture.

(x) **F** Every PDA is equivalent to some DPDA.
The language of palindromes over $\{a, b\}$ is context-free, hence accepted by some DFA, but is not accepted by any DPDA.

(xi) **F** If L has an unambiguous CF grammar, then there must be a DPDA which accepts L.
The language of palindromes over $\{a, b\}$ has an unambiguous CF grammar:
$S \to a|b|\lambda|aSa|bSb$ but is not accepted by any DPDA.

(xii) **T** If Σ is the binary alphabet, Σ^* is countable.
Every language is countable.

(xiii) **F** If Σ is the binary alphabet, the set of languages over Σ is countable.

The set of languages over Σ is equal to the set of subsets of Σ^*. Since Σ^* is infinite, 2^{Σ^*} is uncountable.

(xiv) **O** $\mathcal{P} = \mathcal{NP}$. The most famous open problem in computer science.

2. [10 points] Name a problem which is known to be $\mathcal{NP}$ and is also known to be co-$\mathcal{NP}$, but is not known to be $\mathcal{P}$.

There could be many, but the only one I've told you about is factoring a binary numeral.

3. [10 points] Suppose L is a problem such that you can check any suggested solution in polynomial time. Which one of these statements is certainly true?

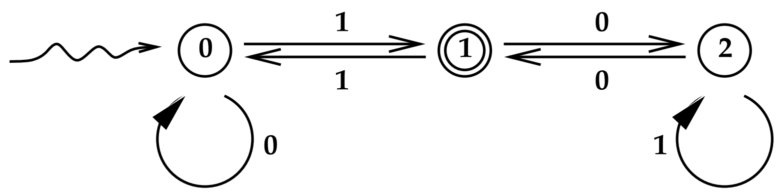
(i) L is $\mathcal{P}$.

(ii) L is $\mathcal{NP}$.

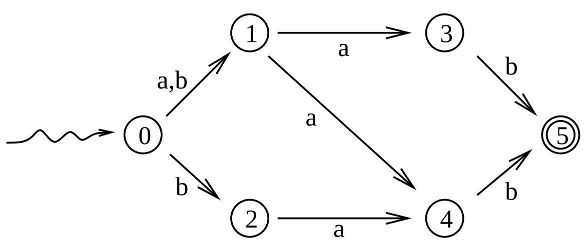
(iii) L is $\mathcal{NP}$-complete.

$\mathcal{NP}$. Verifiability in $\mathcal{P}$ time is the alternative definition of $\mathcal{NP}$. Whether either of the other two answers is correct is unknown. Since I used the word "certainly," the middle choice is the only correct choice.

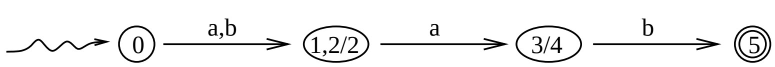
4. [20 points] Let L be the language of all binary strings encoding numbers which are equivalent to 1 modulo 3, where leading zeros are allowed. Thus, $L = \{1, 01, 001, 100, 111, 0100, 0111, 1010, \ldots\}$. Draw a DFA which accepts L. (You need only three states.)



5. [20 points] Consider the NFA M pictured below.



Construct a minimal DFA equivalent to M.



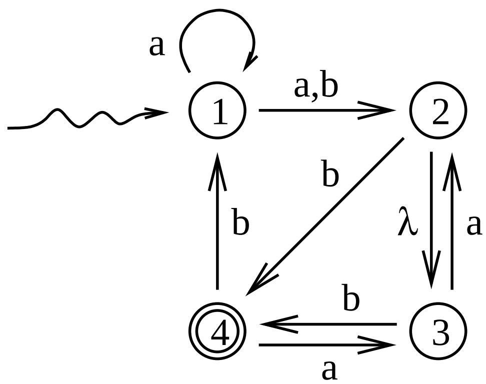
6. [20 points] Let G be the CF grammar given below. Show that G is ambiguous by giving two different rightmost derivations for the string *iiwaea*.

1. $S \to a$
2. $S \to wS$
3. $S \to iS$
4. $S \to iSeS$

$S \Rightarrow iS \Rightarrow iiSeS \Rightarrow iiSea \Rightarrow iiwSea \Rightarrow iiwaea$

$S \Rightarrow iSeS \Rightarrow iSea \Rightarrow iiSea \Rightarrow iiwSea \Rightarrow iiwaea$

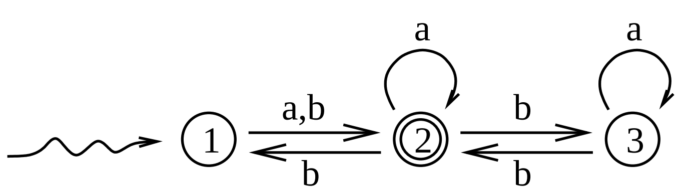
7. [20 points] Give a regular grammar for the language accepted by the follwing NFA.



We will let G have four variables, S, A, B, and C, corresponding to the four states, q_1, q_2, q_3, and q_4.

$S \to aS$
$S \to aA$
$S \to bA$
$S \to aB$
$S \to bB$
$A \to bC$
$A \to aA$
$B \to aA$
$B \to aB$
$B \to bC$
$C \to aB$
$C \to bS$
$C \to \lambda$

8. [20 points] Give a regular expression for the language accepted by the following NFA



The first part gives a direct path to the final state, while the second part gives all loops based at the final state.

$(a + b)(b(a + b) + a + ba^*b)^*$

9. [20 points] State the pumping lemma for regular languages *correctly.* Pay close attention to the order in which you write the quantifiers. If you have all the correct words in the wrong order, you still might get no credit.

Lemma 1 (Pumping Lemma for Regular Languages) *If L is a regular language, there exists a positive integer p, called the pumping length of L, such that for any string $w \in L$ whose length is at least p, there exist strings x, y, z such that the following conditions hold.*

(i) $w = xyz$

(ii) $|y| \geq 1$

(iii) $|xy| \leq p$

(iv) for any $i \geq 0$, $xy^iz \in L$.

10. [20 points] Let L be the language over $\{a, b\}$ generated by the following context-free grammar:
$S \to aSbS$
$S \to \lambda$
Design a PDA which accepts L. Note that L is just the Dyck language, where a and b are used instead of left and right parentheses. That is, each member of L is a string which has the same number of a's as b's, each prefix of which has at least as many a's as b's. That is, $L = \{\lambda, ab, aabb, abab, aaabbb, aababb, abaabb, \ldots\}$