1. Let $M_1$ be the DFA shown below.

Let $M_2$ be the DFA shown below.

Let $M_3$ be the DFA shown below.

Which of the following languages is accepted by $M_1$? By $M_2$? By $M_3$?

(a) The language of all binary strings in which every substring 00 is followed by 1.
(b) All strings over \{a, b\} which end in b and which do not contain the substring bb.
(c) The language of all binary numerals for positive integers equivalent to 2 modulo 3.
(d) The language of all strings over \{a, b\} in which every b is followed by a.

Construct a DFA which accepts the language \{b^i a b^j : i, j \geq 0\}, the language of all strings over \{a, b\} which contain exactly one a. Your figure need not show the dead state.
2. Recall that $\emptyset$ is the empty language. If $L$ is some language, what is the concatenation $\emptyset L$?

3. Let $L_1 = \{\lambda\}$ the language consisting of only the empty string. If $L_2$ is some other language, what is the concatenation $L_1L_2$?

4. Is concatenation of languages commutative? That is, is the equation $L_1L_2 = L_2L_1$ always true?

5. Is it true that, for any language, $L^nL = L^{n+1}$?

6. Which of the following is true:
   
   (a) If $L$ is any language, $L^0 = L$.
   (b) If $L$ is any language, $L^0 = \emptyset$.
   (c) If $L$ is any language, $L^0 = \{\lambda\}$.

   Hint: Think!

7. Does concatenation of languages distribute over union? That is, is $L_1(L_2 + L_3) = L_1L_2 + L_1L_3$ always true?

8. What is $\emptyset^*$, the Kleene closure of the empty language?

9. What is $L^{**}$?

10. Is the union of two regular languages always regular?

11. Is the intersection of two regular languages always regular?

12. Is the complement of a regular language always regular?

13. Is the Kleene closure of a regular language always regular?

14. The DFA $M_1$ shown in Problem 1 is not minimal, that is, it’s equivalent to a DFA with fewer states. Can you draw a state diagram of that DFA? Your figure need not show the dead state.