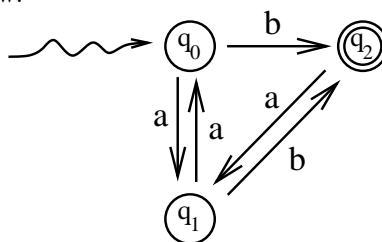
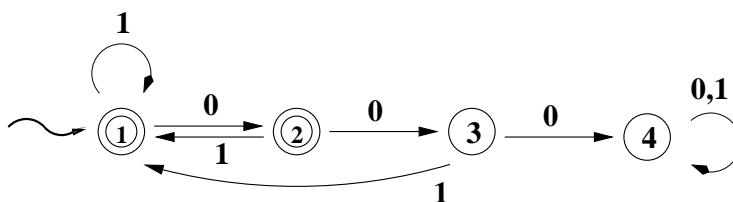


Answers to Assignment 1: Due Friday September 1, 2023

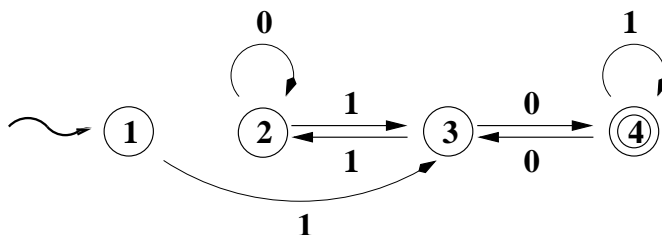
1. Let  $M_1$  be the DFA shown below.



Let  $M_2$  be the DFA shown below.



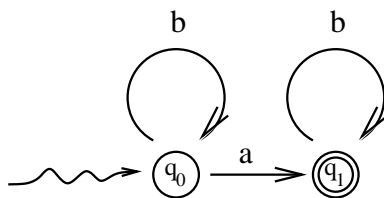
Let  $M_3$  be the DFA shown below.



Which of the following languages is accepted by  $M_1$ ? By  $M_2$ ? By  $M_3$ ?

- (a) I made a mistake drawing the state diagram for  $M_2$ . State 3 is not supposed to be final. The language of all binary strings in which every substring 00 is followed by 1 is not accepted by the original  $M_2$ , but is if we change state 3 to be non-final.
- (b) The language of all strings over  $\{a, b\}$  which end in b and which do not contain the substring bb is accepted by  $M_1$ .
- (c) The language of all binary numerals for positive integers equivalent to 2 modulo 3 is accepted by  $M_3$ .
- (d) The language of all strings over  $\{a, b\}$  in which every b is followed by a is not accepted by any of the machines shown.

Construct a DFA which accepts the language  $\{b^i a b^j : i, j \geq 0\}$ , the language of all strings over  $\{a, b\}$  which contain exactly one  $a$ . Your figure need not show the dead state.



2. Recall that  $\emptyset$  is the empty language. If  $L$  is some language, what is the concatenation  $\emptyset L$ ? Ans:  $\emptyset$
3. Let  $L_1 = \{\lambda\}$ . the language consisting of only the empty string. If  $L_2$  is some other language, what is the concatenation  $L_1 L_2$ ? Ans:  $L_2$
4. Is concatenation of languages commutative? That is, is the equation  $L_1 L_2 = L_2 L_1$  always true? Ans: No.
5. Is it true that, for any language,  $L^n L = L^{n+1}$ ? Ans: Yes.
6. Which of the following is true:
  - (a) If  $L$  is any language,  $L^0 = L$ .
  - (b) If  $L$  is any language,  $L^0 = \emptyset$ .
  - (c) If  $L$  is any language,  $L^0 = \{\lambda\}$ .
 Hint: Think! Ans: False, False, True.
7. Does concatenation of languages distribute over union? That is, is  $L_1(L_2 + L_3) = L_1 L_2 + L_1 L_3$  always true? Ans: Yes.
8. What is  $\emptyset^*$ , the Kleene closure of the empty language? Ans:  $\{\lambda\}$ .
9. What is  $L^{**}$ ? Ans:  $L^*$ . Kleene closure is idempotent.
10. Is the union of two regular languages always regular? Ans: Yes.
11. Is the intersection of two regular languages always regular? Ans: Yes.
12. Is the complement of a regular language always regular? Ans: Yes.
13. Is the Kleene closure of a regular language always regular? Ans: Yes.
14. The DFA  $M_1$  shown in Problem 1 is not minimal, that is, it's equivalent to a DFA with fewer states. Can you draw a state diagram of that DFA? Your figure need not show the dead state.

