University of Nevada, Las Vegas Computer Science 456/656 Fall 2023

Answers to Assignment 4: Due Friday October 13, 2023

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   I expect every student to score 100% on all these questions. Search the internet if you have to.

   (a) **T** All sliding block problems are $\mathcal{P}$–SPACE.

   (b) **T** The game RUSH HOUR is $\mathcal{P}$–SPACE complete.

   (c) **T** The set of binary numerals for prime numbers is a $\mathcal{P}$–TIME language.

   (d) **O** The prime factors of an integer can be computed in $\mathcal{P}$ time if the integer is represented as a binary numeral.

   (e) **T** The prime factors of an integer can be computed in $\mathcal{P}$ time if the integer is represented as a unary numeral.

   (f) **F** Every PDA is equivalent to some DPDA.

   (g) **T** Every language is countable.

   (h) **T** The set of binary languages is uncountable.

   (i) **T** If $\mathcal{P} = \mathcal{NP}$, RSA encryption can be broken in polynomial time.

   (j) **T** Any context-free language over the unary alphabet is regular.

   (k) **T** The complement of any decidable language is decidable.

   (l) **T** The complement of any undecidable language is undecidable.

   (m) **T** Boolean satisfiability is $\mathcal{NP}$-complete.

   (n) **T** If L is any $\mathcal{NP}$ language, there is a $\mathcal{P}$ time reduction of L to Boolean satisfiability.

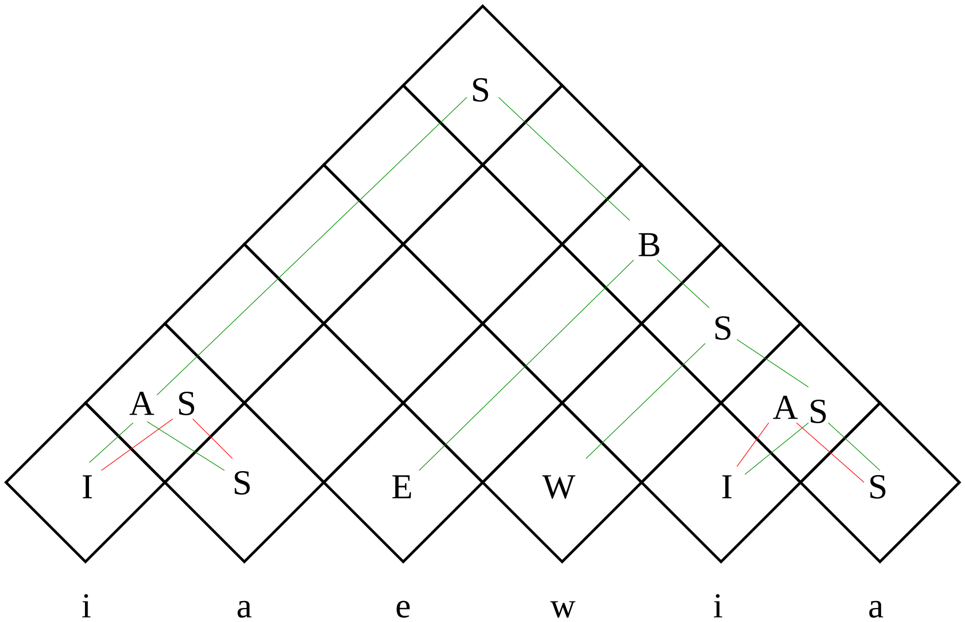
   (o) **T** The language of all contradictions is co-$\mathcal{NP}$.
   A *contradiction* is a Boolean expression if no assignment to the variables makes it true. For example, "x and not x" is a contradiction.

   (p) **O** $\mathcal{P}$–TIME= $\mathcal{P}$–SPACE.

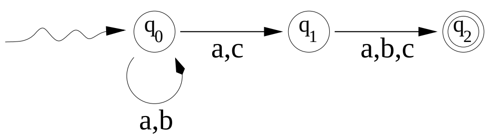
   (q) **T** There is a mathematical statement which is true but has no proof.

2. Let G be the CNF grammar given below. Use the CYK algorithm to show that the $iaewia \in L(G)$.

1. $S \to a$
2. $S \to WS$
3. $W \to w$
4. $S \to IS$
5. $S \to AB$
6. $A \to IS$
7. $B \to ES$
8. $E \to e$
9. $I \to i$

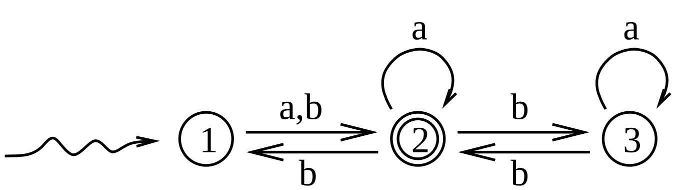


3. Give a grammar, with at most 2 variables, for the language accepted by the follwing NFA.



$S \to aS|bS|cS|aA|cA$
$A \to a|b|c$

4. Give a regular expression for the language accepted by the following NFA



$$(a + b)(b(a + b) + a + ba^*b)^*$$

Which of the following strings are in the language described by the regular expression

$$(a + bc^*)(de^*(d + \lambda) + fg + h)^*$$

- $abcdefgh$ No
- $bdhfghde$ Yes
- $deedhh$ No

5. Let L be the language consisting of all strings over $\{a, b\}$ which have equal numbers of each symbol. Give a CFG for L.

   Here is the simplest grammar I know of

   $S \to aSbS|bSaS|\lambda$

   There are infinitely many correct answers. This problem is theoretically impossible to grade correctly, since the CFG equivalence problem is undecidable. However, If the grader can't figure out whether it's correct within a reasonable time, she will mark it wrong.

6. Let $L = \{a^n b^n c^n \, : \, n \geq 0\}$. As I mentioned in class, L is not context-free. Prove that the complement of L is context-free.

   Let L' be the complement of L. Writing a CFG for L' is lengthy and messy. My proof will be based on the following known facts.

   (a) A regular language is context-free.
   (b) The union of finitely many context-free languages is context-free.
   (c) The concatenation of context-free languages is context-free.

   Let $\Sigma = \{a, b, c\}$. A string w over Σ is not in L for at least one of the following reasons, which we call "flaws."
   • w contains the substring ba, ca, or cb.
   • $w = a^i b^j c^k$, but i, j, k are not all equal.
   L' is the union of languages each of which consists of all strings over Σ which have one of these flaws. We show that each of those languages is context-free.

   Let R be the language consisting of all strings over Σ in which the symbols are not in alphabetical order. R is described by the regular expression $(a + b + c)^*(ba + ca + cb)(a + b + c)^*$, hence R is regular, hence context-free.

   Now, Let
   $X = \{a^i b^j c^k \, : \, i > j\}$
   $Y = \{a^i b^j c^k \, : \, i < j\}$
   $Z = \{a^i b^j c^k \, : \, j > k\}$
   $W = \{a^i b^j c^k \, : \, j < k\}$

   Let $A = \{a\}^*$, $B = \{b\}^*$, and $C = \{c\}^*$. and let $P = \{a^n b^n\}$ and $Q = \{b^n c^n\}$. A, B, C, P, and Q are context-free.

   Then $X = A\{a\}PC$, $Y = P\{b\}BC$, $Z = AB\{b\}Q$, and $W = AQ\{c\}C$, each the concatenation of context-free languages, hence context-free.

   Finally, $L' = R + X + Y + Z + W$, the union of five context-free languages, hence context-free.

7. We say a set S is *proper* if $S \notin S$. Let P be the set of all proper sets. Is P proper? (Hint: Bertrand Russell.)

   You can easily prove that P is proper and also prove that P is not proper. This is the Russell Paradox. Go to the internet to read about the paradox and how it is resolved.