University of Nevada, Las Vegas
Computer Science 456/656 Fall 2022
Answers to Final Examination December 14, 2022

The entire examination is 340 points.

Name:____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

Do not leave the room until you have seriously attempted every problem!

Think! Think! Think! In some cases, the problem was not explicitly mentioned in class or homework, but follows easily from something that was mentioned. No problem is beyond the scope of a student who learned the material.

Definitions you will need. We say that a set S of integers is *recursive* if the set of binary numerals for members of S is recursive, while we say that S is *recursively enumerable* if the set of binary numerals for members of S is recursively enumerable.

1. True/False/Open

   (i) **F** Every subset of a regular language is decidable.

   (ii) **T** The intersection of any two $\mathcal{NP}$ languages is $\mathcal{NP}$.

   (iii) **T** Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (iv) **O** $\mathcal{NC} = \mathcal{P}$.

   (v) **O** $\mathcal{P} = \mathcal{NP}$.

   (vi) **O** The Boolean Circuit Problem (CVP) is in $\mathcal{NC}$.

   (vii) **O** The independent set problem is $\mathcal{P}$-TIME.

   (viii) **T** IF L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

   (ix) **T** If S is a recursive set of positive integers, then $\sum_{n\in S} 2^{-n}$ must be a recursive real number.

   (x) **T** Multiplication of matrices with binary numeral entries is $\mathcal{NC}$.

   (xi) **T** Equivalence of regular expressions is decidable.

   (xii) **T** Every recursively enumerable language is generated by a general grammar.

   (xiii) **T** Equivalence of context-free grammars is co–$\mathcal{RE}$.

   (xiv) **T** The language consisting of all fractions whose values are less than the natural logarithm of 5.0 is recursive.

(xv) **T** If L is in $\mathcal{RE}$ and also co–$\mathcal{RE}$, then L must be decidable.

(xvi) **F** For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(xvii) **F** The language of all true mathematical statements is recursively enumerable.

(xviii) **T** Every sliding block problem is $\mathcal{P}$–SPACE.

(xix) There are uncountably many co–$\mathcal{RE}$ languages.

This problem is incorrectly stated. An alphabet must be specified, in which case the answer is **F**.

(xx) **T** If L is any $\mathcal{P}$–TIME language, there is an $\mathcal{NC}$ reduction of L to CVP, the Boolean circuit problem.

(xxi) **T** There is a polynomial time algorithm for checking whether an integer is prime.

This was open until recently.

(xxii) **T** Every finite language is regular.

(xxiii) **T** If L is a $\mathcal{P}$–TIME language, there is a Turing Machine which decides L in polynomial time.

(xxiv) **T** If anyone ever finds a polynomial time algorithm for any $\mathcal{NP}$–complete language, then $\mathcal{P} = \mathcal{NP}$.

(xxv) **T** RSA encryption is believed to be secure because it is believed that the factorization problem for integers is very hard.

(xxvi) **F** If S is a recursively enumerable set of positive integers, then $\sum_{n\in S} 2^{-n}$ must be a recursive real number.

2. Every language, or problem, falls into exactly one of these categories. For each of the languages, write a letter indicating the correct category. [5 points each]

A Known to be $\mathcal{NC}$.
B Known to be $\mathcal{P}$–TIME, but not known to be $\mathcal{NC}$.
C Known to be $\mathcal{NP}$, but not known to be $\mathcal{P}$–TIME and not known to be $\mathcal{NP}$–complete.
D Known to be $\mathcal{NP}$–complete.
E Known to be $\mathcal{P}$–SPACE but not known to be $\mathcal{NP}$
F Known to be decidable, but not nown to be $\mathcal{P}$–SPACE.
G $\mathcal{RE}$ but not decidable.
H co-$\mathcal{RE}$ but not decidable.
I Neither $\mathcal{RE}$ nor co-$\mathcal{RE}$.

(a) **H** All C++ programs which do not halt if given themselves as input.

(b) **A** All base 10 numerals for perfect squares.

(c) **A** The Dyck language.

(d) **H** $\{\langle G\rangle : L(G)$ is the Dyck language.$\}$

(e) **E** All positions of RUSH HOUR from which it is possible to win.

(f) **D** The Jigsaw problem. (That is, given a finite set of two-dimensional pieces, can they be assembled into a rectangle, with no overlap and no spaces.)

(g) **C** Factorization of binary numerals.

3. [20 points] Find a DFA equivalent to the NFA shown in Figure 1.

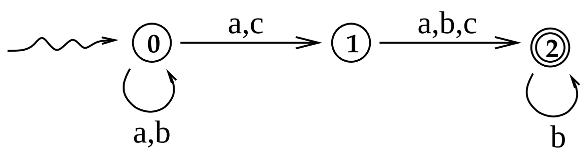


Figure 1: NFA for problems 3 and 4

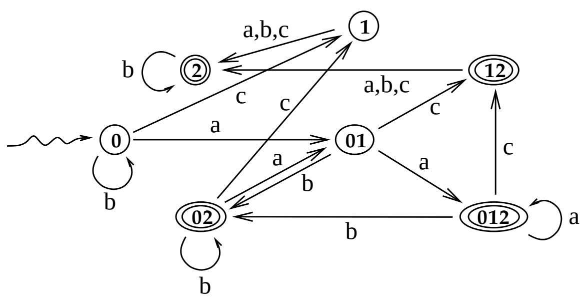


Figure 2: Equivalent DFA

4. [20 points] Give a regular grammar for the language accepted by the machine in Figure 1.

$S \to aS$
$S \to bS$
$S \to aB$
$S \to cB$
$B \to aC$
$B \to bC$
$B \to cC$
$C \to bC$
$C \to \lambda$

5. [20 points] Give a regular expression for the language accepted by the machine in Figure 3

$a^*b(ca^*b + a + ac^*b)^*$

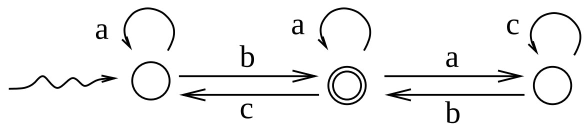


Figure 3: NFA for problem 5.

6. Which class of languages does each of these machine classes accept? [5 points each]

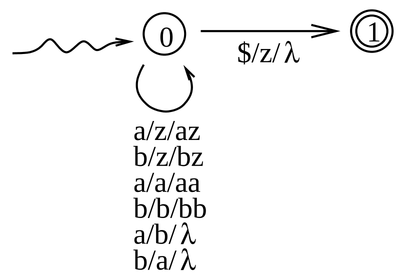
   (a) Deterministic finite automata. **Regular Languages**

   (b) Non-deterministic finite automata. **Regular Languages**

   (c) Push-down automata. **Context-Free Languages**

   (d) Turing Machines. **Recursively Enumerable Languages**

7. [20 points]
   Let $L = \{w \in \{a,b\}^* : \#_a(w) = \#_b(w)\}$, that is, each string of L has equal numbers of each symbol. Draw a PDA which accepts L.



8. [20 points] The grammar below is an ambiguous CF grammar with start symbol E, and is parsed by the LALR parser whose ACTION and GOTO tables are shown here. The ACTION table is missing actions for the second column, when the next input symbol is the "minus" sign. Fill it in. Remember the C++ precedence of operators. (Hint: the column has seven different actions: s2, s4, r1, r2, r3, r4, and r5, some more than once, and has no blank spaces.)

   1. $E \to E -_2 E_3$
   2. $E \to -_4 E_5$
   3. $E \to E *_6 E_7$
   4. $E \to (_8 E_9)_{10}$
   5. $E \to x_{11}$

	x	$-$	$*$	(	)	\$	S
0	$s11$	$s4$		$s8$			1
1		$s2$	$s6$			halt	
2	$s11$	$s4$		$s8$			3
3		$r1$	$s6$		$r1$	$r1$	
4	$s11$	$s4$		$s8$			5
5		$r2$	$r2$		$r2$	$r2$	
6	$s11$	$s4$		$s8$			7
7		$r3$	$r3$		$r3$	$r3$	
8	$s11$	$s4$		$s8$			9
9		$s2$	$s6$		$s6$		
10		$r4$	$r4$	$r4$	$r4$	$r4$	
11		$r5$	$r5$		$r5$	$r5$	

9. [20 points] Prove that any decidable language can be enumerated in canonical order by some machine.

Let $L \subseteq \Sigma^*$ be a decidable language. Let $w_1, w_2, \ldots$ be the canonical order enumeration of Σ^* The following program enumeraes L. in canonical order.

For all i from 1 to ∞
    If $w_i \in L$
        Write w_i

10. [20 points] Give a polynomial time reduction of 3-SAT to to the independent set problem.

Let $E = C_1 * C_2 * \cdots * C_k$ be a Boolean expression in 3-CNF form. Let $C_i = t_{i,1} + t_{i,2} + t_{i,3}$ where each $t_{i,j}$ is either a variable or the negation of a variable. Let G be a graph of $3k$ vertices $V = \{v_{i,p} : 1 \leq i \leq k, 1 \leq p \leq 3\}$

There is an edge between $v_{i,p}$ and $v_{j,q}$ if and only if either $i = j$ or $t_{i,p} * t_{j,q}$ is a contradiction.

11. [20 points] Prove that the halting problem is undecidable.

Prove that the halting problem is undecidable.

By contradiction. Suppose the halting problem is decidable. Let L_{diag} be the set of programs which do not halt if given themselves as input. The following program, which we call P_{diag}, accepts L_{diag}:

read a program P
if P halts with input P
    Enter an infinite loop
else HALT

Note that the condition in the second line of P_{diag} can always be evaluated because the halting problem is decidable.

We now ask, is P_{diag} a member of L_{diag}? The answer cannot be either yes or no. Thus, the halting problem is undecidable.