

CSC 456/656 Spring 2022 Final Examination May 11, 2022

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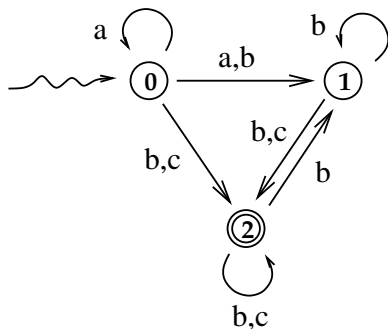
The entire test is 475 points.

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, \mathcal{P} and \mathcal{NP} denote \mathcal{P} -TIME and \mathcal{NP} -TIME, respectively.
 - (i) _____ The language $\{a^i b^j c^k \mid j = i + k\}$ is context-free.
 - (ii) _____ The intersection of any two context-free languages is context-free.
 - (iii) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.
 - (iv) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.
 - (v) _____ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.
 - (vi) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.
 - (vii) _____ Every problem that can be mathematically defined has an algorithmic solution.
 - (viii) _____ The intersection of two undecidable languages is always undecidable.
 - (ix) _____ Every \mathcal{NP} language is decidable.
 - (x) _____ $\mathcal{NC} = \mathcal{P}$.
 - (xi) _____ $\mathcal{P} = \mathcal{NP}$.
 - (xii) _____ The language consisting of all satisfiable Boolean expressions is \mathcal{NP} -complete.
 - (xiii) _____ The Boolean Circuit Problem is in \mathcal{P} .
 - (xiv) _____ The Boolean Circuit Problem is in \mathcal{NC} .
 - (xv) _____ If L_1 and L_2 are undecidable languages, there must be a recursive reduction of L_1 to L_2 .
 - (xvi) _____ 2-SAT is \mathcal{P} -TIME.
 - (xvii) _____ 3-SAT is \mathcal{P} -TIME.
 - (xviii) _____ Primality is \mathcal{P} -TIME.
 - (xix) _____ There is a \mathcal{P} -TIME reduction of the halting problem to 3-SAT.

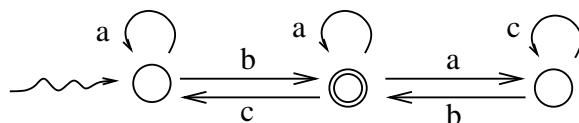
- (xx) ——— Every context-free language is in \mathcal{NC} .
- (xxi) ——— Addition of binary numerals is in \mathcal{NC} .
- (xxii) ——— Every language generated by an unrestricted grammar is recursive.
- (xxiii) ——— For any two languages L_1 and L_2 , if L_1 is undecidable and there is a recursive reduction of L_1 to L_2 , then L_2 must be undecidable.
- (xxv) ——— If P is a mathematical proposition that can be written using a string of length n , and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.
- (xxvi) ——— Every bounded function is recursive.
- (xxvii) ——— If L is \mathcal{NP} and also $\text{co-}\mathcal{NP}$, then L must be \mathcal{P} .
- (xxviii) ——— Recall that if \mathcal{L} is a class of languages, $\text{co-}\mathcal{L}$ is defined to be the class of all languages that are not in \mathcal{L} . Let \mathcal{RE} be the class of all recursively enumerable languages. If L is in \mathcal{RE} and also L is in $\text{co-}\mathcal{RE}$, then L must be decidable.
- (xxix) ——— If a language L is undecidable, then there can be no machine that enumerates L .
- (xxx) ——— There is a non-recursive function which grows faster than any recursive function.
- (xxxi) ——— For every real number x , there exists a machine that runs forever and outputs the string of decimal digits of x .
- (xxxii) ——— Every subset of a regular language is regular.
- (xxxiii) ——— The computer language C++ has Turing power.
- (xxxiv) ——— If L is any \mathcal{P} -TIME language, there is an \mathcal{NC} reduction of L to the Boolean circuit problem.
- (xxxv) ——— The binary integer factorization problem is $\text{co-}\mathcal{NP}$.
- (xxxvi) ——— There is a polynomial time reduction of the subset sum problem to the binary factorization problem.

2. [20 points] Construct a minimal DFA equivalent to the NFA shown below.



3. [20 points] Find an NFA which accepts the language generated by this grammar.
- $$\begin{aligned}
 S &\rightarrow aA|cS|cC \\
 A &\rightarrow aA|bS|cB|\lambda \\
 B &\rightarrow aA|cB|bC|\lambda \\
 C &\rightarrow aB
 \end{aligned}$$

4. [20 points] Give a regular expression which describes the language accepted by this NFA.

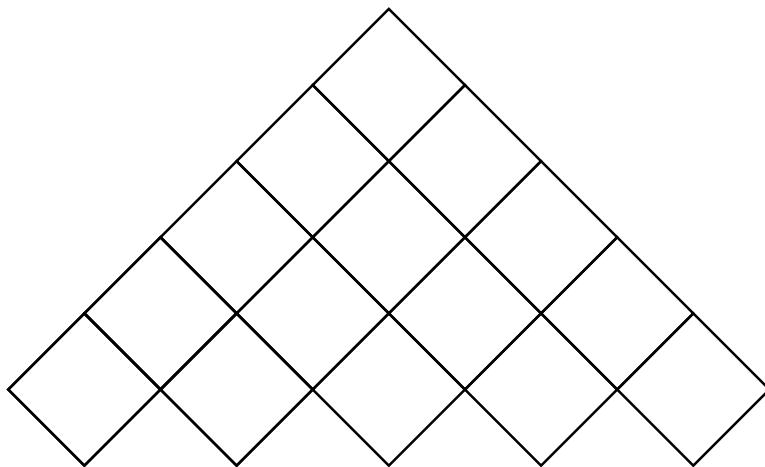


5. [20 points] Construct a PDA which accepts the language $L = \{w \in \{a, b\}^* : \#_a(w) = \#_b(w)\}$, the language of all strings over $\{a, b\}$ which have equal numbers of the two symbols.

6. [20 points] Use the CYK algorithm to decide whether $abcab$ is generated by the CNF grammar:

$$\begin{aligned}
 S &\rightarrow AB|BC|CA \\
 A &\rightarrow a \\
 B &\rightarrow SA|SS|b \\
 C &\rightarrow c
 \end{aligned}$$

by filling in the matrix.



7. [20 points] The LALR parser given for this grammar:

1. $E \rightarrow E -_2 E_3$
2. $E \rightarrow E *_4 E_5$
3. $E \rightarrow x_6$

contains errors, meaning that it might parse a string in a manner that would be considered incorrect by your programming instructor. Find those errors and correct them.

	x	$-$	$*$	$\$$	E
0	$s6$				1
1		$s2$	$s4$	$halt$	
2	$s6$				3
3		$r1$	$r1$	$r1$	
4	$s6$				5
5		$s2$	$s4$	$r2$	
6		$r3$	$r3$	$r3$	

8. [20 points] Prove that the grammar given in Problem 7 is ambiguous by giving two different leftmost derivations for some string. (If you simply give two different parse trees, you have not answered the question.)

9. [20 points] State the pumping lemma for regular languages.

10. [20 points] Fill in the blanks: L is \mathcal{NP} if and only if there is a _____ reduction of _____ to _____, where _____ is \mathcal{NP} -complete.

11. [20 points] Give the verifier-certificate definition of the class \mathcal{NP} .

12. [10 points] What is the importance nowadays of \mathcal{NC} ?
13. [10 points] What complexity class contains sliding block problems?
14. [20 points] Give a polynomial time reduction of the subset sum problem to the partition problem.
15. Label each of the following sets as countable or uncountable.
- (i) ----- The set of integers.
 - (ii) ----- The set of rational numbers.
 - (iii) ----- The set of real numbers.
 - (iv) ----- The set of binary languages.
 - (v) ----- The set of co-RE binary languages.
 - (vi) ----- The set of undecidable binary languages.
 - (vii) ----- The set of functions from integers to integers.
 - (viii) ----- The set of recursive real numbers.

16. [20 points] Prove that the halting problem is undecidable.