CSC 456/656 Answers to Spring Final Examination, May 11, 2022

Name:____

The entire test is 475 points.

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) **T** The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.

   (ii) **T** The intersection of any two context-free languages is context-free.

   (iii) **T** Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (iv) **T** The problem of whether a given string is generated by a given context-free grammar is decidable.

   (v) **F** Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

   (vi) **O** There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (vii) **F** Every problem that can be mathematically defined has an algorithmic solution.

   (viii) **F** The intersection of two undecidable languages is always undecidable.

   (ix) **T** Every $\mathcal{NP}$ language is decidable.

   (x) **O** $\mathcal{NC} = \mathcal{P}$.

   (xi) **O** $\mathcal{P} = \mathcal{NP}$.

   (xii) **T** The language consisting of all satisfiable Boolean expressions is $\mathcal{NP}$-complete.

   (xiii) **T** The Boolean Circuit Problem is in $\mathcal{P}$.

   (xiv) **O** The Boolean Circuit Problem is in $\mathcal{NC}$.

   (xv) **F** If L_1 and L_2 are undecidable langugages, there must be a recursive reduction of L_1 to L_2.

   (xvi) **T** 2-SAT is $\mathcal{P}$-TIME.

   (xvii) **O** 3-SAT is $\mathcal{P}$-TIME.

   (xviii) **T** Primality is $\mathcal{P}$-TIME.

   (xix) **F** There is a $\mathcal{P}$-TIME reduction of the halting problem to 3-SAT.

   (xx) **T** Every context-free language is in $\mathcal{NC}$.

(xxi) **T** Addition of binary numerals is in $\mathcal{NC}$.

(xxii) **F** Every language generated by an unrestricted grammar is recursive.

(xxiii) **T** For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(xxv) **F** If P is a mathematical proposition that can be written using a string of length n, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

(xxvi) **F** Every bounded function is recursive.

(xxvii) **O** If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(xxviii) **T** If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

(xxix) **F** If a language L is undecidable, then there can be no machine that enumerates L.

(xxx) **T** There is a non-recursive function which grows faster than any recursive function.

(xxxi) **T** There exists a machine that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(xxxiii) **F** For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(xxxiv) **F** Every subset of a regular language is regular.

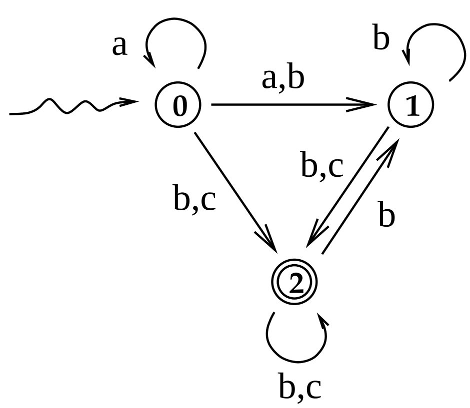
(xxxv) **T** The computer language C++ has Turing power.

(xxxvi) **T** If L is any $\mathcal{P}$-TIME language, there is an $\mathcal{NC}$ reduction of L to the Boolean circuit problem.

(xxxvii) **T** The binary integer factorization problem is co-$\mathcal{NP}$.

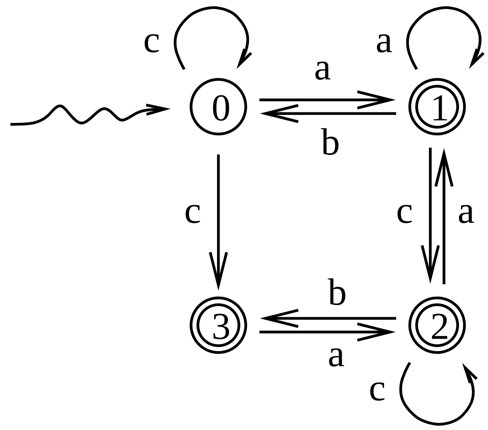
(xxxviii) **O** There is a polynomial time reduction of the subset sum problem to the binary factorization problem.

2. [20 points] Construct a minimal DFA equivalent to the NFA shown below.

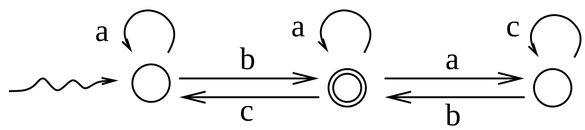


3. [20 points] Find an NFA which accepts the language generated by this grammar.

$S \to aA|cS|cC$
$A \to aA|bS|cB|\lambda$
$B \to aA|cB|bC|\lambda$
$C \to aB$



4. [20 points] Give a regular expression which describes the language accepted by this NFA.



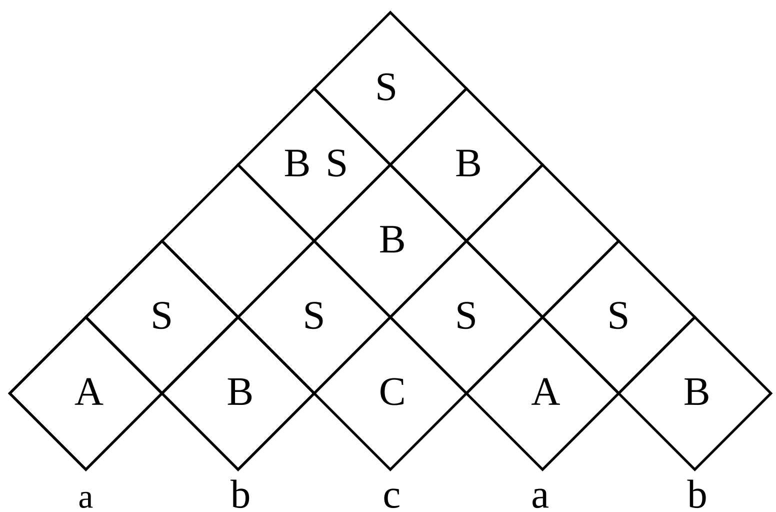
$$a^*b(ca^*b + a + ac^*b)^*$$

5. [20 points] Construct a PDA which accepts the language $L = \{w \in \{a,b\}^* : \#_a(w) = \#_b(w)\}$, the language of all strings over $\{a,b\}$ which have equal numbers of the two symbols.

6. [20 points] Use the CYK algorithm to decide whether $abcab$ is generated by the CNF grammar:

$S \to AB \,|\, BC \,|\, CA$
$A \to a$
$B \to SA \,|\, SS \,|\, b$
$C \to c$

by filling in the matrix.



7. [20 points] The LALR parser given for this grammar:

1. $E \to E -_2 E_3$
2. $E \to E *_4 E_5$
3. $E \to x_6$

contains errors, meaning that it might parse a string in a manner that would be considered incorrect by your programming instructor. Find those errors and correct them.

	x	$-$	$*$	$\$$	E
0	$s6$				1
1		$s2$	$s4$	*halt*	
2	$s6$				3
3		$r1$	$s4$	$r1$	
4	$s6$				5
5		$r2$	$s4$	$r2$	
6		$r3$	$r3$	$r3$	

8. [20 points] Prove that the grammar given in Problem 7 is ambiguous by giving two different leftmost derivations for some string. (If you simply give two different parse trees, you have not answered the question.)

$E \Rightarrow E - E \Rightarrow x - E \Rightarrow x - E * E \Rightarrow x - x * E \Rightarrow x - x * x$
$E \Rightarrow E * E \Rightarrow E - E * E \Rightarrow x - E * E \Rightarrow x - x * E \Rightarrow x - x * x$

9. [20 points] State the pumping lemma for regular languages.

For any regular language L
there is an integer p such that
for any $w \in L$ of length at least p
there exist strings x, y, z such that the following four statements hold

1. $w = xyz$
2. $|xy| \le p$
3. $|y| \ge 1$
4. for any $i \ge 0$, $xy^iz \in L$

10. [20 points] Fill in the blanks.

L is $\mathcal{NP}$ if and only if there is a $\mathcal{P}$–TIME reduction of L to L_2, where L_2 is $\mathcal{NP}$–complete.

11. [20 points] Give the verifier-certificate definition of the class $\mathcal{NP}$.

A language L is $\mathcal{NP}$ if and only if there exists a determinisitc machine V and an integer k such that

(i) For any $w \in L$ there exists a string c (a *certificate* for w) such that V accepts the string $w\#c$ in $O(n^k)$ time, where $n = |w|$

(ii) If $w \notin L$ and c is any string, V does not accept the string $w\#c$. That is, there is no certificate for any string not in L.

12. [10 points] What is the importance nowadays of $\mathcal{NC}$?

Machines with many processors have become more common, and it is important to consider which problems can be solved with programs that make efficient use of that parallel structure.

13. [10 points] What complexity class contains sliding block problems?

$\mathcal{P}$–SPACE

14. [20 points] Give a polynomial time reduction of the subset sum problem to the partition problem.

Let $\alpha = (x_1, x_2, \ldots x_n, K)$ be an instance of the subset sum problem. We reduce α to an instance β of the partition problem which has a solution if and only if α has a solution.

Let $S = \sum_{i=1}^{n}$. If $S < K$, α has no solution, so we let $\beta = (1)$, an instance of the partion problem that, trivially, has no solution. Otherwise, we let $\beta = (x_1, x_2, \ldots x_n, K+1, S-K+1)$.

15. Label each of the following sets as countable or uncountable.

(i) The set of integers. Countable.

(ii) The set of rational numbers. Countable.

(iii) The set of real numbers. Uncountable.

(iv) The set of binary languages.

(v) . Uncountable. The set of co-RE binary languages. Countable.

(vi) The set of undecidable binary languages. Uncountable.

(vii) The set of functions from integers to integers. Uncountable.

(viii) The set of recursive real numbers. Countable.

16. [20 points] Prove that the halting problem is undecidable.

By contradiction. Suppose the halting problem is decidable. Let L_{diag} be the set of programs which do not halt if given themselves as input. The following program, which we call P_{diag}, accepts L_{diag}:

```
read a program P
if P halts with input P
    Enter an infinite loop
else HALT
```

Note that the condition in the second line of P_{diag} can always be evaluated because the halting problem is decidable.

We now ask, is P_{diag} a member of L_{diag}? The answer cannot be either yes or no. Thus, the halting problem is undecidable.