CSC 456/656 Fall 2023 First Examination Problems to Study

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) **F** Every subset of a regular language is regular.

   (ii) **T** The class of regular languages is closed under intersection.

   (iii) **O** $\mathcal{P}$–TIME = $\mathcal{NP}$.

   (iv) **T** The class of regular languages is closed under Kleene closure.

   (v) **T** The class of context-free languages is closed under union.

   (vi) **F** The class of context-free languages is closed under intersection.

   (vii) **F** The set of binary numerals for prime numbers is a regular language.

   (viii) **T** The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

   (ix) **F** The complement of any context-free language is context-free.

   (x) **T** The complement of any recursive (that is, decidable) language is recursive.

   (xi) **T** If Σ is an alphabet, then Σ^* is a regular language.

   (xii) **F** If L is a language and L^* is a regular language, then L must be a regular language. **(Think!)**

   (xiii) **T** The class of languges which are **not regular** is closed under intersection. **(Think!)**

   (xiv) **F** A minimal DFA equivalent to an NFA with n states must have 2^n states.

   (xv) **O** If a non-derministic machine can solve a given problem in polynomial time, then there is a deterministic machine which can solve the same problem in polynomial time.

   (xvi) **T** If a non-derministic machine can solve a given problem in polynomial time, then there is a deterministic machine which can solve the same problem in exponential time.

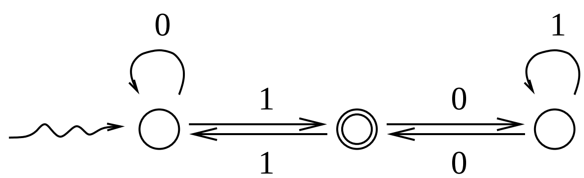
2. Give an example of a language which is context-free but not regular.

   $\{a^n b^n\}$

3. Give an example of a language which is not context-free.

   $\{a^n b^n c^n\}$

4. Let L be the language of all binary strings encoding numbers which are equivalent to 1 modulo 3, where leading zeros are allowed. Thus, $L = \{1, 01, 001, 100, 111, 0100, 0111, 1010, \ldots\}$. Draw a DFA which accepts L. (You need only three states.)



5. Let G be the CF grammar given below, where E is the start symbol. Show that G is ambiguous by giving two different **rightmost** derivations for the string $x - y * z$.

   1. $E \to E - E$
   2. $E \to E * E$
   3. $E \to x$
   4. $E \to y$
   5. $E \to z$

   $E \Rightarrow E - E \Rightarrow x - E \Rightarrow x - E * E \Rightarrow x - y * E \Rightarrow x - y * z$

   $E \Rightarrow E * E \Rightarrow E - E * E \Rightarrow x - E * E \Rightarrow x - y * E \Rightarrow x - y * z$

   The first derivation is the one that respects the usual precedence of the operators.

6. Give a grammar for the language accepted by the NFA shown in Figure 1 below.

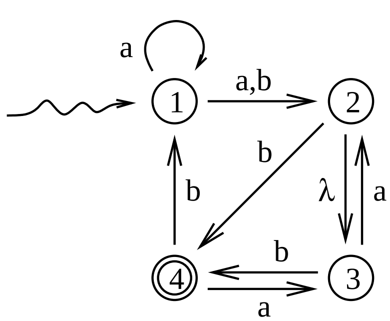
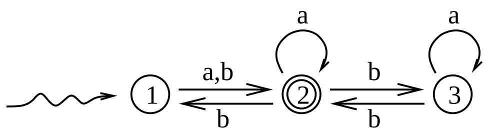


Figure 1: NFA for problems 6 and 9.

$S \to aS$
$S \to aA$
$S \to bA$
$A \to B$
$A \to bC$
$B \to bC$
$B \to aA$
$C \to aB$
$C \to bS$
$C \to \lambda$

7. Write a regular expression for the language accepted by the following NFA



$(a+b)(a+b(a+b)+ba*b)*$

8. Write the pumping lemma for regular languages *correctly.* Pay close attention to the order in which you write the quantifiers. If you have all the correct words in the wrong order, you still might get no credit. **This will not be on the September 27 examination.**

For any regular language L, there is an integer p (which is called the pumping length of L) such that

For any $w \in L$ such that $|w| \geq p$

There exist strings x, y, z such that the following four statements hold:

1. $w = xyz$
2. $|barredxy \leq p$
3. $|barredy \geq 1$
4. for any integer $i \geq 0$, $xy^iz \in L$

9. Draw a minimal DFA equivalent to the NFA shown in Figure 1 in problem 6 above. Show the transition table, and also show the matrix used for minimizing the DFA.

	a	b
1	123	23
2	23	4
3	23	4
((4	3	1
123	123	234
23	23	4
((234	23	14
((14	123	234

	1	2	3	4	123	23	234	14
1	O	X	X	X	X	X	X	X
2	X	O	O	X	X	O	X	X
3	X	O	O	X	X	O	X	X
4	X	X	X	O	X	X	X	X
123	X	X	X	X	O	X	X	X
23	X	O	O	X	X	O	X	X
234	X	X	X	X	X	X	O	X
14	X	X	X	X	X	X	X	O