CSC 456/656 Fall 2023 Answers to First Examination September 27, 2023

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) **F** Every subset of a regular language is regular.
   Every language is the subset of some regular language.

   (ii) **F** The complement of a CFL is always a CFL.

   (iii) **T** The class of context-free languages is closed under union.

   (iv) **F** The class of context-free languages is closed under intersection.

   (v) **T** The set of binary numerals for multiples of 23 is regular.
   The set of numerals (of any base, not just 2) for the members of any arithmetic sequence is a regular language.

   (vi) **T** The set of binary numerals for prime numbers is in $\mathcal{P}$–TIME.
   The base doen't matter, as long as its at least 2. (This excludes unary (caveman) numerals.) This is a fact that was proven only recently, by Maninda Agrawal, N. Kayal, and N. Saxena, and published in 2004, but I believe the result leaked out ealier. Before then, the correct answer to this question would have been **O**.

   (vii) **F** Every PDA is equivalent to some DPDA.
   See Problem 2 below.

   (viii) **T** Every language is countable.
   There are only countably many strings over any given alphabet.

   (ix) **F** The set of languages over the binary alphabet is countable.
   Let Σ be any alphabet. Then Σ^* is the set of all strings over Σ, which is infinite and countable. But Cantor proved that, for any set S, the set 2^S has more elements than S. The set of all languages over any alphabet Σ is 2^{Σ^*}, which is then not countable.

   (x) **O** $\mathcal{P} = \mathcal{NP}$.
   Solve this and you will be **really** famous.

   (xi) **T** The complement of any $\mathcal{P}$–TIME language is $\mathcal{P}$–TIME.
   If a machine decides a language L, it (by switching the 0 and 1 outputs) decides the complement of L in the same number of steps. This rule does not hold for acceptance.

   (xii) **O** The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.
   If $\mathcal{P} = \mathcal{NP}$, then the answer is true, otherwise it is false, so it's open.

(xiii) **T** The complement of any decidable language is decidable.

If a machine decides a language L, it (by switching the 0 and 1 outputs) decides the complement of L.

(xiv) **T** The complement of any undecidable language is undecidable. let L' be the complement of L. If L is undecidable and L' is decidable, this violates the answer to the previous question.

2. Give an unambiguous CFG which generates a language not accepted by any DPDA.

   There are many correct answers, but I believe the one given here is simplest.
   $S \to aSa$
   $S \to bSb$
   $S \to \lambda$

3. Suppose L is a problem such that you can check any suggested solution in polynomial time. Which one of these statements is certainly true?

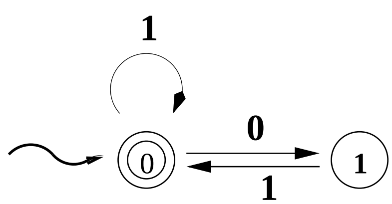
   (i) L is $\mathcal{P}$.

   (ii) L is $\mathcal{NP}$.

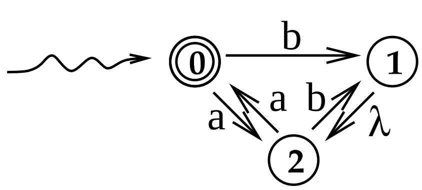
   (iii) L is $\mathcal{NP}$-complete.

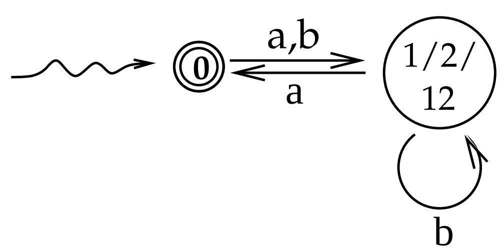
   Only the second one. If $\mathcal{P} = \mathcal{NP}$, all three statements are equivalent, hence true.

4. L be the language of all binary strings in which each 0 is followed by 1. Draw a DFA which accepts L.



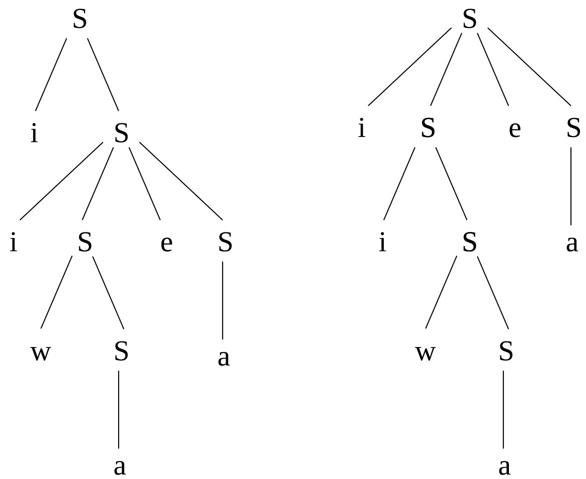
5. Consider the NFA M pictured below. Construct a minimal DFA equivalent to M.





6. Let G_1 be the CF grammar given below. Prove that G_1 is ambiguous by giving two different parse trees for the string $iiwaea$.

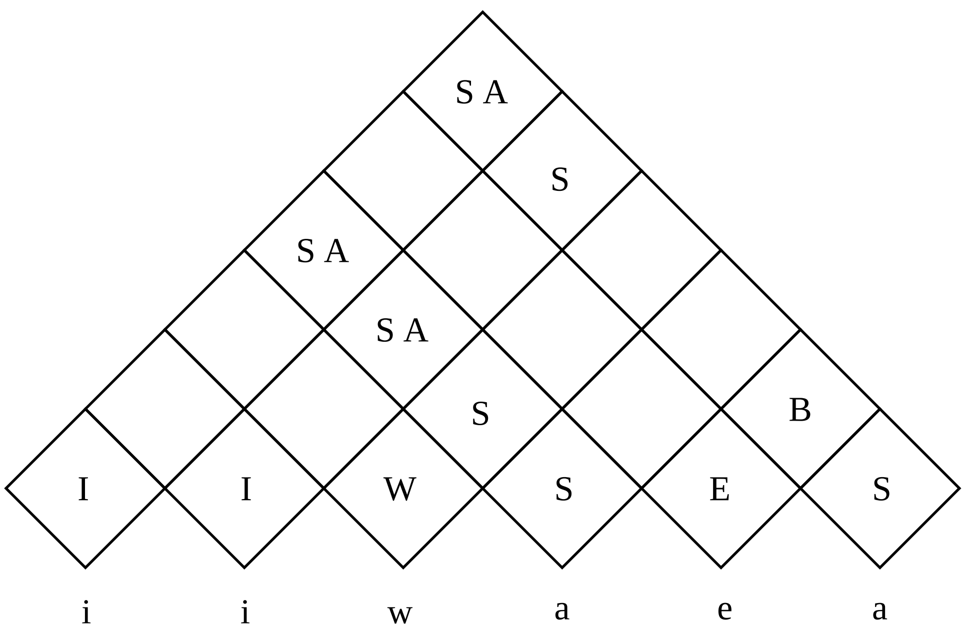
1. $S \to a$
2. $S \to wS$
3. $S \to iS$
4. $S \to iSeS$



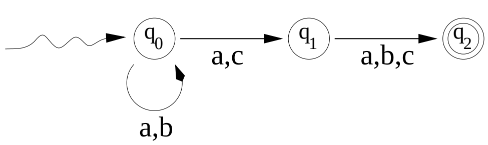
7. The CNF grammar G_2, given below, is equivalent to the grammar G_1 given in Problem 6.

   Use the CYK algorithm to prove that $iiwaea$ is generated by G_2.

1. $S \to a$
2. $S \to WS$
3. $W \to w$
4. $S \to IS$
5. $S \to AB$
6. $A \to IS$
7. $B \to ES$
8. $E \to e$



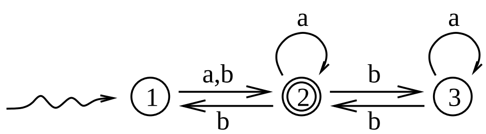
8. Give a grammar, with at most 3 variables, for the language accepted by the follwing NFA.



$S \to aS|bS|aA|cA$
$A \to aB|bB|cB$
$B \to \lambda$

You actually need only two variables. Do you see how?

9. Give a regular expression for the language accepted by the following NFA



$(a + b)(b(a + b) + a + ba^*b)^*$

10. Let L be the language consisting of all strings over $\{a, b\}$ which have equal numbers of each symbol. Give a CFG for L.

    There are many solutions. The grammar below is, I believe, the simplest. It is ambiguous, but L does have an unambigous CFG.

    $S \to aSbS$
    $S \to bSaS$
    $S \to \lambda$

11. Design a DPDA which accepts the language described in Problem 10.