University of Nevada, Las Vegas Computer Science 456/656 Fall 2024

Assignment 3:

This assignment will not be graded. Answers will be posted on Tuesday September 23, 2024.

$\mathcal{P}$ means $\mathcal{P}$–TIME.

1. True/False. If the answer is not known to science at this time, enter "O" for Open.

   (i) _____ co-$\mathcal{P} = \mathcal{P}$.

   (ii) _____ co-$\mathcal{NP} = \mathcal{NP}$.

   (iii) _____ co-$\mathcal{P}$–SPACE $= \mathcal{P}$–SPACE.

   (iv) _____ Block placement problems are $\mathcal{NP}$.

   (v) _____ Sliding block problems are $\mathcal{P}$–SPACE.

   (vi) _____ $\mathcal{P}$–SPACE $= \mathcal{NP}$

   (vii) _____ Regular expression equivalence is $\mathcal{P}$.

   (viii) _____ Regular expression equivalence is decidable.

   (ix) _____ Context-free grammar equivalence is decidable.

   (x) _____ Every regular language is context-free.

   (xi) _____ The language C++ is context-free.

   (xii) _____ The intersection of any two context-free languages is context-free.

   (xiii) _____ The complement of any context-free language is context-free.

   (xiv) _____ Every language is countable.

   (xv) _____ For any real number x, there is a program that prints the decimal expansion of x.

   (xvi) _____ For any real number x, there is a machine that decides whether a fraction is less than x.

   (xvii) _____ There are only countably many decidable binary languages.

   (xviii) _____ Given a regular grammar G with n variables, there exists an NFA with n variables that accepts $L(G)$.

   (xix) _____ $\left\{ a^i b^j c^k : i = k \right\}$ is a context-free language.

   (xx) _____ Given an integer n written in binary notation, it is possible to find the prime factors of n in polynomial time.

   (xxi) _____ Given an integer n written in binary notation, it is possible to decide whether n is prime in polynomial time.

   (xxii) _____ Any language generated by a grammar is decidable.

   (xxiii) _____ The complement of any decidable language is decidable.

   (xxiv) _____ The union of any two decidable languages is decidable.

   (xxv) _____ The complement of any undecidable language is undecidable.

   (xxvi) _____ The union of any two undecidable languages is undecidable.

   (xxvii) _____ Every context-free language is accepted by some DPDA.

2. Let L be the language generated by the following CNF (Chomsky Normal Form) grammar.

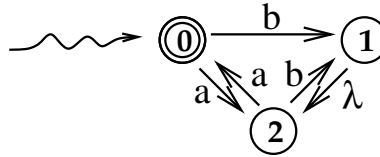$S \to AS \mid CD \mid e$

$C \to AS$

$D \to BS$

$A \to a$

$B \to b$

Use the CYK algorithm to

determine whether $aaebae \in L$.

3. Give a context-free grammar for $L = \left\{ w \in \{a,b\}^* : \#_a(w) > \#_b(w) \right\}$, that is, strings which have more a's than b's.

4. Write a regular grammar which generates the language accepted by the NFA illustrated below.

5. List the grammar classes and language classes of the Chomsky hierarchy.

6. Give two context-free languages whose intersection is not context-free.

7. Write a grammar for the Dyck language (using 'a' and 'b' instead of parentheses) and give a derivation of the string abaabb.

8. Draw a PDA which accepts the Dyck language, using a and b instead of left and right parentheses, respectively.

9. In the following, do not write more than necessary. Your answers should be concise and correct.

(a) Explain the verification definition of the class $\mathcal{NP}$.

(b) What could be a certificate to prove that a given Boolean expression is in the language SAT?