

University of Nevada, Las Vegas Computer Science 456/656 Fall 2024

Assignment 4: Due October 13, 2024, 11:59 PM

Thu Oct 10 09:26:39 AM PDT 2024

Name: _____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from the graduate assistant, Sabrina Wallace, telling you how to turn in the assignment.

\mathcal{P} means \mathcal{P} -TIME.

1. True/False. If the answer is not known to science at this time, enter "O" for Open.
 - (i) _____ $\text{co-}\mathcal{P} = \mathcal{P}$.
 - (ii) _____ $\text{co-}\mathcal{NP} = \mathcal{NP}$.
 - (iii) _____ $\text{co-}\mathcal{P}\text{-SPACE} = \mathcal{P}\text{-SPACE}$.
 - (iv) _____ Block placement problems are \mathcal{NP} .
 - (v) _____ Sliding block problems are $\mathcal{P}\text{-SPACE}$.
 - (vi) _____ $\mathcal{P}\text{-SPACE} = \mathcal{NP}$
 - (vii) _____ Regular expression equivalence is \mathcal{P} .
 - (viii) _____ Regular expression equivalence is decidable.
 - (ix) _____ Context-free grammar equivalence is decidable.
 - (x) _____ Every regular language is context-free.
 - (xi) _____ The language C++ is context-free.
 - (xii) _____ The intersection of any two context-free languages is context-free.
 - (xiii) _____ The complement of any context-free language is context-free.
 - (xiv) _____ Every language is countable.
 - (xv) _____ For any real number x , there is a program that prints the decimal expansion of x .
 - (xvi) _____ For any real number x , there is a machine that decides whether a fraction is less than x .
 - (xvii) _____ There are only countably many decidable binary languages.
 - (xviii) _____ Given a regular grammar G with n variables, there exists an NFA with n variables that accepts $L(G)$.
 - (xix) _____ $\{a^i b^j c^k : i \neq j \text{ or } j \neq k\}$ is a context-free language.
 - (xx) _____ Given an integer n written in binary notation, it is possible to find the prime factors of n in polynomial time.
 - (xxi) _____ Given an integer n written in binary notation, it is possible to decide whether n is prime in polynomial time.
 - (xxii) _____ Any language generated by a grammar is decidable.
 - (xxiii) _____ The complement of any decidable language is decidable.

- (xxiv) ----- The union of any two decidable languages is decidable.
- (xxv) ----- The complement of any undecidable language is undecidable.
- (xxvi) ----- The union of any two undecidable languages is undecidable.
- (xxvii) ----- Every context-free language is accepted by some DPDA.
- (xxviii) ----- If some machine writes an increasing sequence of fractions which converges to x , then x must be a recursive real number.
2. State the pumping lemma for regular languages. If your answer contains all the right words, but not in the right order, you might get no credit.

3. Draw a PDA which accepts the language $L = \{a^i b^j c^k : i = j \text{ or } j = k\}$

4. List the names (not the definitions) of three \mathcal{NP} -complete problems (languages) that we have **not** discussed in class.

5. Given languages L_1 and L_2 , exactly one of the following statements is correct. Which one? -----

- (i) If there is an easy reduction from L_1 to L_2 and L_1 is hard, then L_2 must be hard.
- (ii) If there is an easy reduction from L_1 to L_2 and L_2 is hard, then L_1 must be hard.

- (iii) If there is an easy reduction from L_1 to L_2 and L_1 is easy, then L_2 must be easy.
 - (iv) If there is a hard reduction from L_1 to L_2 and L_2 is easy, then L_1 must be easy.
6. Explain the verification definition of the class \mathcal{NP} . Do not write more than necessary. Your answer should be concise and correct.

7. Prove that, for any positive integer n , the sum of the first n cubes, $1^3 + 2^3 + 3^3 + \dots + n^3$, is $\frac{n^2(n+1)^2}{4}$

8. Prove that $\sqrt{3}$ is irrational.

9. Prove that $\log_2 3$ is irrational. (Hint: What is the definition of logarithm?)

10. However, $\log_2 3$ is very close to the rational number $19/12$, only about 1% off. Explain why this fact is important for Western music.¹

¹From the internet: “Western music may be defined as organized instrumentation and sound created and produced in Europe, the United States, and other societies established and shaped by European immigrants. This includes a wide assortment of musical genres, from classical music and jazz to rock and roll and country-western music.”