

University of Nevada, Las Vegas Computer Science 456/656 Fall 2024
Assignment 6: Due Saturday November 9, 2024, 11:59 PM

Name: _____

You are permitted to work in groups, get help from others, read books, and use the internet.

1. True/False/Open

- (i) _____ Every regular language is \mathcal{NC} .
 - (ii) _____ Every context-free language is \mathcal{NC} .
 - (iii) _____ $\mathcal{NC} = \mathcal{P-TIME}$
 - (iv) _____ Given polynomially many processors, the product of two n -bit binary numerals can be computed in polylogarithmic time. _____
 - (v) The asymptotic time complexity of a sequential algorithm cannot be less than _____ its asymptotic space complexity.
 - (vi) _____ The asymptotic space complexity of a sequential algorithm cannot be less than its asymptotic time complexity.
 - (vii) _____ In fourth grade (or whenever) you learned how to multiply two n -digit decimal numerals in $O(n^2)$ time and space. But in the best computers nowadays, multiplication of integers is worked faster, thanks to parallel circuitry and microprogramming.
 - (viii) _____ Matrix multiplication with integral entries is \mathcal{NC} .
 - (ix) _____ Every finite language is decidable.
 - (x) _____ If a theorem can be stated using n bits of text and is provably true, then it must have a proof which has length polynomial in n .
2. Give a $\mathcal{P-TIME}$ reduction of the Subset Sum problem to Partition. This time, **learn** it, instead of trying to remember it verbatim.

3. Give a \mathcal{P} -TIME reduction of 3-SAT to the Independent Set problem. This time, **learn** it, instead of trying to remember it verbatim.

4. Use the pumping lemma to prove that $L = \{a^n b^n : n \geq 0\}$ is not regular.

5. State the Church Turing thesis.

6. Prove that $\sqrt{3}$ is irrational.

7. (a) Give a definition of an *instance* of the halting problem.

(b) Prove that the halting problem is undecidable.

- (c) On the handout, I gave a proof that the halting problem is decidable. Of course, this proof is incorrect. The flaw in that proof is that I made a hidden (not explicitly stated) assumption that is false. What is that hidden assumption?

8. If you are parallelizing a sequential algorithm, it would be desirable to create a parallel algorithm with polylogarithmic time complexity, but without increasing the total work. There is an obvious sequential algorithm for the maxarray problem, *i.e.*, finding the maximum of an array of n numbers, which takes $O(n)$ time. In class I gave you a logarithmic time algorithm for the same problem which uses $O(n)$ processors. Thus, its work complexity is $O(n \log n)$, which is greater than for the sequential algorithm.

Find a parallel algorithm for the maxarray problem whose time complexity is $O(\log n)$ and whose work complexity is $O(n)$.

This problem is hard. It is not hard to describe, but it is hard to think of. I am not sure anyone will get it. (Hint: it's "out there.")