$\mathcal{P}$ and $\mathcal{NP}$

We write $\mathcal{N}$ to mean the *natural numbers*, the numbers you learned in pre-school: the numbers people used before anyone invented negative numbers, rational numbers, real numbers, complex numbers, and even zero. In other words, the positive integers.

A *polynomial function* is a function $f : \mathcal{N} \to \mathcal{N}$ such that, for some positive integers k and N, $f(n) \leq n^k$ for all $n \geq N$.

We say that a language $L \subseteq \Sigma^*$ is a member of the class $\mathcal{P} - time$ if there is a machine M which decides L in polynomial time. More specifically: there is a deterministic machine M which takes any string $w \in \Sigma^*$ as input, and which outputs 1 if $w \in L$ and 0 if $w \notin L$, and which runs in at most $f(n)$ steps, where f is a polynomial function and $n = |w|$, the length of w.