CSC 456/656 Fall 2024 Answers to Examination September 25, 2024

The entire exam is 230 points.

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of members of $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) **F** Every subset of a regular language is regular.

   (ii) **F** The complement of a CFL is always a CFL.

   (iii) **T** The class of context-free languages is closed under union.

   (iv) **F** The class of context-free languages is closed under intersection.

   (v) **T** The set of binary numerals for prime numbers is $\mathcal{P}$–TIME.

   (vi) **F** The set of languages over the binary alphabet is countable.

   (vii) **O** $\mathcal{P} = \mathcal{NP}$.

   (viii) **T** The complement of any $\mathcal{P}$–TIME language is $\mathcal{P}$–TIME.

   (ix) **O** The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

   (x) **T** The complement of any decidable language is decidable.

   (xi) **T** The complement of any undecidable language is undecidable.

   (xii) **O** If L is both $\mathcal{NP}$ and co–$\mathcal{NP}$, then L must be $\mathcal{P}$–TIME.

   (xiii) **T** If L is both $\mathcal{RE}$ and co–$\mathcal{RE}$, then L must be decidable.

   (xiv) **T** There are countably many recursive real numbers.

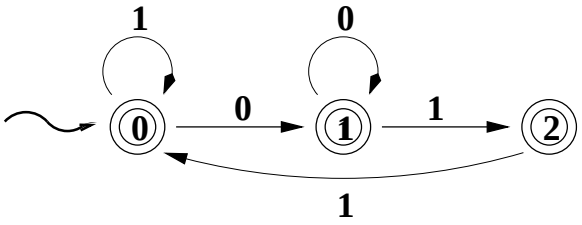
   (xv) **F** If a sequence of fractions converges to a real number x, then x must be recursive.

   (xvi) **T** Every context-free language is $\mathcal{P}$–TIME.

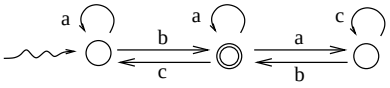
   (xvii) **T** The halting problem is $\mathcal{RE}$.

   (xviii) **F** The CFG equivalence problem is $\mathcal{RE}$.

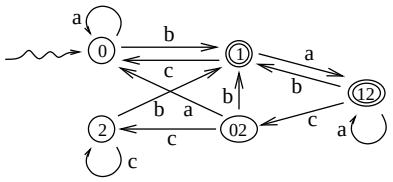
2. [20 points] L be the language of all binary strings which do not have the substring 010. Draw a DFA which accepts L.



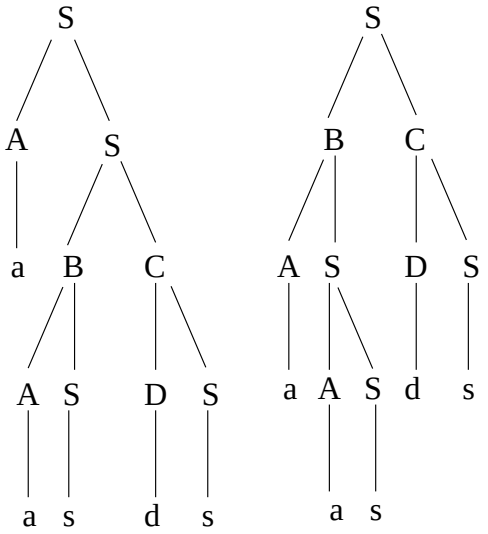
3. [20 points] Consider the NFA M pictured below. Construct a minimal DFA equivalent to M.



	a	b	c
0	0	1	$\emptyset$
1	12	$\emptyset$	0
2	$\emptyset$	1	2
12	12	1	02
02	0	1	2



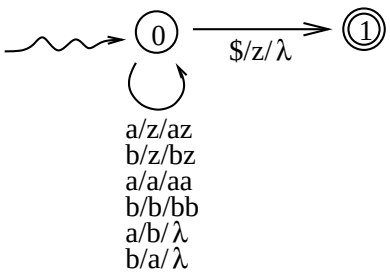
4. [20 points] Let G be the CF grammar given below. Prove that G is ambiguous by giving two different parse trees for the string $aasds$.
   1. $S \to s$
   2. $S \to AS$
   3. $A \to a$
   4. $S \to BC$
   5. $B \to AS$
   6. $C \to DS$
   7. $D \to d$



5. [20 points] Let L be the language consisting of all strings over $\{a, b\}$ which have equal numbers of each symbol. Give a CFG for L.

   $S \to aSbS$
   $S \to bSaS$
   $S \to \lambda$

6. [20 points] Draw a PDA which accepts the language given in problem 5.



7. [20 points] Give a regular expression for the language accepted by the NFA shown in problem 3.

   $a^*b(ca^*b + a + ac^*b)^*$

8. [20 points] Let L be the set of binary numerals for negative positive integers which are equivalent to 1 mod 3. (n such that $n\%3 = 1$) That is, $L = \{1, 100, 111, 1010, 1101, \ldots\}$. Draw a DFA that accepts L.