CSC 456/656 Spring 2024 First Examination February 7 2024

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want a scratch page to be graded, write "See scratch paper," on the test, and write your name on the scratch paper.

The entire examination is 240 points.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. (5 points each)

    (i) _____ Every subset of a regular language is regular.

    (ii) _____ The class of regular languages is closed under intersection.

    (iii) _____ $\mathcal{P}$–TIME $= \mathcal{NP}$.

    (iv) _____ The class of regular languages is closed under Kleene closure.

    (v) _____ The class of context-free languages is closed under union.

    (vi) _____ The class of context-free languages is closed under intersection.

    (vii) _____ The set of binary numerals for prime numbers is a regular language.

    (viii) _____ The Kleene closure of the empty language is empty.

    (ix) _____ The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

    (x) _____ The complement of any context-free language is context-free.

    (xi) _____ The complement of any recursive (that is, decidable) language is recursive.

    (xii) _____ If Σ is an alphabet, then Σ^* is a regular language.

    (xiii) _____ If L is a language and L^* is a regular language, then L must be a regular language.

    (xiv) _____ In regular expressions, concatenation distributes over union.

    (xv) _____ The regular grammar equivalence problem is decidable.

    (xvi) _____ The context-free grammar equivalence problem is decidable.

    (xvii) _____ The regular expression equivalence problem is decidable.

    (xviii) _____ The language of palindromes over $\{a, b\}$ is not accepted by any PDA.

    (xix) _____ Any context-free language is generated by some unambiguous context-free grammar.

    (xx) _____ The Dyck language is context-free.

(xxi) _____ The complement of $L = \{a^n b^n c^n : n \geq 0\}$ is context-free. (We never did this in class, but if you think, you can figure it out using material we have covered.)

(xxii) _____ Every language is accepted by some machine.

(xxiii) _____ Let $F(n)$ be the nth decimal digit of π after the decimal point. Then F is recursive.

2. [5 points] A language is context-free if and only if it is accepted by some _____.

3. [5 points] Give an example of a language which is context-free but not regular.

4. [5 points] Give an example of a language which is not context-free.

5. [20 points] Let G be the CF grammar given below, where S is the start symbol. G is ambiguous by giving two different **leftmost** derivations (not parse trees) for some string generated by G.

1. $S \rightarrow iS$
2. $S \rightarrow iSeS$
3. $S \rightarrow wS$
4. $S \rightarrow a$

6. [15 points] Given an unambiguous context-free grammar for the Dyck language, where (for ease of grading) you write "a" insted of "(" and "b" instead of ")" Partial credit for an ambiguous grammar.

7. [15 points] Write a regular expression for the language accepted by the machine shown below.
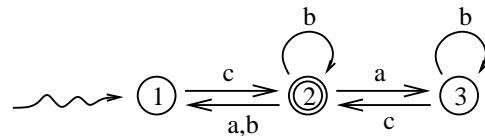
8. [20 points] What are the four language (or grammar) classes of the Chomsky hierarchy? Be sure to mention the type numbers as well as the name of the class.

9. [20 points] Find a minimal DFA equivalent to the NFA shown below.
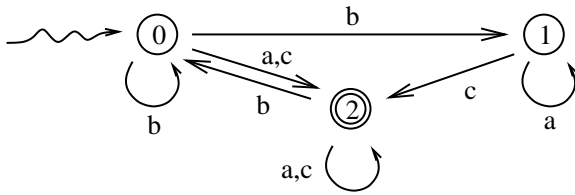
10. (a) [15 points] What language does the DFA below accept? (Hint: Arithmetic property.)

(b) [15 points] Write a regular expression for that language.