

Computer Science 456/656 Fall 1998 Midterm, October 15, 1998

Name: _____

No books, notes, or scratch paper. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 175 points.

1. True or False. [5 points each]

- (a) _____ Any subset of a regular language is a regular language.
- (b) _____ The intersection of any two context-free languages is context-free.
- (c) _____ The language $\{a^i b^i : i \geq 0\}$ does not have an ambiguous grammar.
- (d) _____ The set of all regular expressions over the alphabet $\{0,1\}$ is a regular language.

2. Fill in each blank with **one** word. [5 points each blank]

- (a) Every NFA with n states is equivalent to a unique _____ DFA, which has at most _____ states. (I want the formula.)
- (b) The intersection of any CFL with any _____ language is a CFL.

3. What does the CKY algorithm do? [10 points]. Just answer the question, do not give an example.

4. Draw a minimal DFA which accepts the language of all strings over $\{0,1\}$ which contain at least two instances of 01 as a substring. [15 points]

5. Let G be the CFG whose productions are given below:

$$S \rightarrow \Lambda$$

$$S \rightarrow aSb$$

$$S \rightarrow bSa$$

$$S \rightarrow SS$$

(a) Prove that all strings in $L(G)$ have equal numbers of a 's and b 's. [10 points]

(b) Prove that all strings over $\{a, b\}$ which have equal numbers of a 's and b 's are in $L(G)$. [15 points]

6. Let $L_1 = \{a^i b^i a^i : i \geq 0\}$. Let L_2 be the complement of L_1 , *i.e.*, all strings over $\{a, b\}$ which are not in L_1 . Prove that L_2 is not a CFL, or prove that it is. [30 points]

7. Consider the grammar G whose terminal alphabet is $\{(\ , \), +, -, a, b\}$, whose variables alphabet is $\{S, T, V\}$, and whose productions are:

$$S \rightarrow S + T$$

$$S \rightarrow S - T$$

$$S \rightarrow T$$

$$T \rightarrow V$$

$$T \rightarrow (S)$$

$$V \rightarrow aV$$

$$V \rightarrow bV$$

$$V \rightarrow a$$

$$V \rightarrow b$$

- (a) Construct a parse tree for the string $a - b - ab - (bb + ab - aa)$. [15 points]

- (b) Construct a CNF grammar equivalent to G . [15 points]

8. Let L be the set of all strings over $\{a, b\}$ which have equal numbers of a 's and b 's. Use the pumping lemma to prove that L is not regular. [30 points]