Computer Science 456/656 Fall 1998 Midterm, November 24, 1998

Name:____

No books, notes, or scratch paper. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 200 points.

1. True or False. [5 points each]
   (a) _______ A language is said to be *recursive* if it has a recursive enumeration.
   (b) _______ Any subset of a regular language is a regular language.
   (c) _______ Every recursively enumerable language can be generated by a general grammar.
   (d) _______ Any partial function that can be computed by any machine can be computed by some Turing Machine.
   (e) _______ Every Turing Machine can be emulated by a 2-PDA, defined as a machine just like a PDA, but with two stacks instead of just one.
   (f) _______ The complement of every recursive language is recursive.
   (g) _______ The complement of every recursively enumerable language is recursively enumerable.
   (h) _______ The following problem is decidable: given any general grammar G and any string w, does G generate w?
   (i) _______ If $L = L(T)$, where T is an NTM (non-deterministic Turing machine), then L is recursively enumerable.

2. Fill in each blank with **one** word. [5 points each blank]
   (a) A language is said to be *undecidable* if it is not ____________________________.
   (b) A machine M is said to be ____________________________ if for every configuration ξ of M there is at most one configuration of M which follows from ξ in one step.
   (c) A ______________ is a machine that inputs a string of a language generated by a given grammar and outputs a derivation of that string.

3. Let $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, .\}$. Let L be the language over Σ consisting of all strings which are prefixes of the decimal expansion of the well-known real number π. (For example, L includes the strings $\Lambda, 3, 3., 3.1, 3.14, 3.141, \ldots$) Prove that L is recursive. [10 points]

4. Informally, describe how a multi-track single tape TM can be emulated by a one-track single tape TM. [15 points]

5. Informally, describe how a multi-tape TM, where each tape has a single track, can be emulated by a multi-track single tape TM. [15 points]

6. Design a parser for the following grammar: [30 points] Hint: your parser may be any of the types that we studied in class.

   1. $S \rightarrow s$
   2. $S \rightarrow aSSb$
   3. $S \rightarrow cSdS$

7. Use the pumping lemma to prove that the language $L = \{a^n b^n a^n\}$ is not context-free. [20 points]

8. Suppose that L is a language. For each integer n, define $P_n(L)$ to be the set of all strings in L whose lengths are at most n.

   Suppose that, for each $n \geq 0$, $P_n(L)$ is recursive. Prove that L is recursive. [20 points]

9. If T is a TM, our textbook uses the notation $e(T)$ to denote the *encoding* of T. In my lecture on the board, I used the notation "T" to mean the same thing as $e(T)$.

Suppose that $f : \mathcal{N} \to \mathcal{N}$ is a function which has the following property. If T is a TM whose encoding $e(T)$ has length m, and if w is any string accepted by T, and if $|w| = \ell$, then T accepts w in at most $f(m + \ell)$ steps.

(By the way, such a function f really exists. That fact is easily proved, but I am not asking you for that proof.)

Prove that f is not recursive. (Hint: use the fact that the diagonal language, as defined in class, is not recursive.) [30 points]