Computer Science 456/656. Problem 1 of Assignment 9, Spring 2014.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (i) _____ There exists a machine[1] that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

   (ii) _____ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

   (iii) _____ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

   (iv) _____ The problem of whether two given context-free grammars generate the same language is decidable.

   (v) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (vi) _____ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   (vii) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (viii) _____ There exists a mathematical proposition that can be neither proved nor disproved.

   (ix) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$.

   (x) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (xi) _____ Every undecidable problem is $\mathcal{NP}$-complete.

   (xii) _____ The problem of whether a given context-free grammar generates all strings is decidable.

   (xiii) _____ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

   (xiv) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (xv) _____ The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.

   (xvi) _____ The intersection of any three regular languages is context-free.

   (xvii) _____ If a language L is undecidable, then there can be no machine that enumerates L.

   (xviii) _____ **(Warning: this one is hard.)** If f is any function on positive integers, there must be a recursive function g such that $f(n) = O(g(n))$.

[1] As always in automata theory, "machine" means abstract machine, a mathematical object whose memory and running time are **not** constrained by the size and lifetime of the known (or unknown) universe, or any other physical laws. If we want to discuss the kind of machine that exists (or could exist) physically, we call it a "physical machine."

(xix) Recall that if $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$.

_____ Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

(xx) _____ Every problem that can be mathematically defined has an algorithmic solution.

(xxi) _____ The intersection of two undecidable languages is always undecidable.

(xxii) _____ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(xxiii) _____ If L is a context-free language over an alphabet with just one symbol, then L is regular.

(xxiv) _____ Every $\mathcal{NP}$ language is decidable.

(xxv) _____ The intersection of two $\mathcal{NP}$ languages must be $\mathcal{NP}$.

(xxvi) _____ The intersection of two $\mathcal{NP}$-complete languages must be $\mathcal{NP}$-complete.

(xxvii) _____ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

(xxviii) _____ If G is a context-free grammar, the question of whether $L(G) = \Sigma^*$ is decidable, where Σ is the terminal alphabet of G.

(xxix) _____ The set of all fractions whose values are less than π is decidable. (Think of a fraction as a string of digits, followed by a slash, followed by another string of digits.)

(xxx) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary numeral.

(xxxi) _____ Every bounded function is recursive.

(xxxii) _____ For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(xxxiii) _____ For any two languages L_1 and L_2, if L_2 is undecidable and there is a recursive reduction of L_1 to L_2, then L_1 must be undecidable.

(xxxiv) _____ If P is a mathematical proposition that can be stated using n binary bits, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

(xxxv) _____ Let L be the language consisting of just one string of length 1, defined as follows:

$$L = \begin{cases} \{1\} \text{ if } \mathcal{P} = \mathcal{NP} \\ \{0\} \text{ if } \mathcal{P} \neq \mathcal{NP} \end{cases}$$

Then L is undecidable.

(xxxvi) _____ There is a $\mathcal{P}$-TIME reduction of the context-free grammar membership problem to the halting problem.