

T/F/O: Known to be true, Known to be false, or Open.

1. \_\_\_\_\_ Every subset of a regular language is regular.
2. \_\_\_\_\_ Let  $L$  be the language over  $\Sigma = \{a, b\}$  consisting of all strings of the form  $a^m b^n$ , where  $m \geq n$ . Then  $L$  is a regular language.
3. \_\_\_\_\_ The complement of every regular language is regular.
4. \_\_\_\_\_ If a language has an context-free grammar, then it is is accepted by some push-down automaton.
5. \_\_\_\_\_ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.
6. \_\_\_\_\_ There is a PDA that accepts all valid C++ programs.
7. \_\_\_\_\_ The intersection of any two regular languages is regular.
8. \_\_\_\_\_ The language consisting of all base 3 numerals for positive integers  $n$  such that  $n \% 7 = 3$  is regular.
9. \_\_\_\_\_ The language consisting of all base 5 numerals for prime positive integers.
10. \_\_\_\_\_ The intersection of any two context-free languages is context-free.
11. \_\_\_\_\_ The Kleene closure of every context-free language is context-free.
12. \_\_\_\_\_ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.
13. \_\_\_\_\_ There is a PDA that accepts all valid C++ programs.
14. \_\_\_\_\_  $\mathcal{P}$ -TIME =  $\mathcal{NP}$ -TIME.
15. \_\_\_\_\_ If a languages is not in the class  $\mathcal{P}$ -TIME, then it must be undecidable.
16. \_\_\_\_\_ The set of all mathematical statements which are provably true is decidable.
17. \_\_\_\_\_ The set of all mathematical statements which are provably true is recursively enumerable.
18. \_\_\_\_\_ If a language  $L$  is accepted by some NTM, then there is a TM which accepts  $L$ .
19. \_\_\_\_\_ The problem of whether a given grammar generates the empty language is decidable.
20. \_\_\_\_\_ The problem of whether a given grammar generates all strings over an alphabet  $\Sigma$  is decidable.
21. \_\_\_\_\_ Every language that can be parsed by an LALR parser can be accepted by some DPDA.
22. \_\_\_\_\_ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.
23. \_\_\_\_\_ If a real number  $x$  is the solution to a polynomial equation with integral coefficients, then there must be a TM that runs forever, writing the decimal digits of  $x$ .

24. \_\_\_\_\_ There exists a polynomial time algorithm which finds the factors of any positive integer, if the input integer is written in unary (“caveman”) notation.
25. \_\_\_\_\_ The Boolean circuit problem is undecidable.
26. \_\_\_\_\_ The complement of every recursive language is recursive.
27. \_\_\_\_\_ The complement of every recursively enumerable language is recursively enumerable.
28. \_\_\_\_\_ Every language which is generated by an unrestricted grammar is recursively enumerable.
29. \_\_\_\_\_ The question of whether two context-free grammars generate the same language is undecidable.
30. \_\_\_\_\_ There exists some proposition which is true but which has no proof.
31. \_\_\_\_\_ The set of all binary numerals for prime numbers is in the class  $\mathcal{P}$ .
32. \_\_\_\_\_ If  $L_1$  reduces to  $L_2$  in polynomial time, and if  $L_1$  is  $\mathcal{NP}$ , and if  $L_2$  is  $\mathcal{NP}$ -complete, then  $L_1$  must be  $\mathcal{NP}$ -complete.
33. \_\_\_\_\_ Given any context-free grammar  $G$  and any string  $w \in L(G)$ , there is always a unique leftmost derivation of  $w$  using  $G$ .
34. \_\_\_\_\_ For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.
35. \_\_\_\_\_ Using multi-processors and other advanced technology, it is possible to design a machine which decides the halting problem.
36. \_\_\_\_\_ The question of whether two regular expressions are equivalent is decidable.
37. \_\_\_\_\_ Let  $L = \{ \langle M \rangle \mid M \text{ halts with no input} \}$ . Then  $L$  is recursively enumerable.
38. \_\_\_\_\_ The complement of every context-free language is context-free.
39. \_\_\_\_\_ The complement of every  $\mathcal{NP}$  language is  $\mathcal{NP}$ .
40. \_\_\_\_\_ The class of context-free languages is a subclass of  $\mathcal{P}$ .
41. \_\_\_\_\_ No language which has an ambiguous context-free grammar can be accepted by a DPDA.
42. \_\_\_\_\_ The question of whether a given Turing Machine halts with empty input is decidable.
43. \_\_\_\_\_ The class of languages accepted by non-deterministic finite automata is the same as the class of languages accepted by deterministic finite automata.
44. \_\_\_\_\_ If  $L_1$  reduces to  $L_2$  in polynomial time, and if  $L_2$  is  $\mathcal{NP}$ , then  $L_1$  must be  $\mathcal{NP}$ .
45. \_\_\_\_\_ Let  $F(0) = 1$ , and let  $F(n) = 2^{F(n-1)}$  for  $n > 0$ . Then  $F$  is Turing-computable.

46. \_\_\_\_\_ Every language which is accepted by some non-deterministic machine is accepted by some deterministic machine.
47. \_\_\_\_\_ The language of all regular expressions over the binary alphabet is a regular language.
48. \_\_\_\_\_ Let  $\pi$  be the ratio of the circumference of a circle to its diameter. (That's the usual meaning of  $\pi$  you learned in second grade.) The problem of whether the  $n^{\text{th}}$  digit of  $\pi$ , for a given  $n$ , is equal to a given digit is decidable.
49. \_\_\_\_\_ There cannot exist any computer program that can decide whether any two C++ programs are equivalent.
50. \_\_\_\_\_ An undecidable language is necessarily  $\mathcal{NP}$ -complete.
51. \_\_\_\_\_ Every context-free language is in the class  $\mathcal{P}$ -TIME.
52. \_\_\_\_\_ Every function that can be mathematically defined is Turing computable.
53. \_\_\_\_\_ The language of all binary strings which are the binary numerals for prime numbers is context-free.
54. \_\_\_\_\_ Every bounded function from integers to integers is Turing-computable. (We say that  $f$  is *bounded* if there is some  $B$  such that  $|f(n)| \leq B$  for all  $n$ .)
55. \_\_\_\_\_ The language of all palindromes over  $\{0, 1\}$  is inherently ambiguous.
56. \_\_\_\_\_ Every context-free grammar can be parsed by some deterministic top-down parser.
57. \_\_\_\_\_ Every context-free grammar can be parsed by some non-deterministic top-down parser.
58. \_\_\_\_\_ Every context-free grammar can be parsed by some deterministic bottom-up parser.
59. \_\_\_\_\_ Every context-free grammar can be parsed by some non-deterministic bottom-up parser.
60. \_\_\_\_\_ Commercially available parsers cannot use the LALR technique, since most modern programming languages are not context-free.
61. \_\_\_\_\_ The boolean satisfiability problem is undecidable.
62. \_\_\_\_\_ There is a parallel processor machine which can solve the boolean circuit problem in polylogarithmic time.
63. \_\_\_\_\_ The set of all binary numerals for prime numbers is in the class  $\mathcal{P}$ -TIME.
64. \_\_\_\_\_ If  $\mathcal{P} = \mathcal{NP}$ , then RSA coding is insecure.
65. \_\_\_\_\_ The Boolean satisfiability problem is  $\mathcal{NP}$ -complete.

Fill in the blanks.

1. Name two classes of machines that accept the class of regular languages. \_\_\_\_\_  
and \_\_\_\_\_.

2. Name one class of machines that accepts the class of context-free languages. \_\_\_\_\_
3. If a machine  $M$  is \_\_\_\_\_, there is at most one legal move  $M$  can make from any give configuration.
4. Name two classes of machines that accept the class of regular languages. \_\_\_\_\_  
and \_\_\_\_\_.
5. Name one class of machines that accepts the class of context-free languages. \_\_\_\_\_
6. If a machine  $M$  is \_\_\_\_\_, there is at most one legal move  $M$  can make from any give configuration.
7. An LALR parser outputs a derivation of its input string which corresponds to the \_\_\_\_\_ visitation of the internal nodes of its parse tree.
8. If every string generated by a grammar  $G$  has a unique leftmost derivation using  $G$ , then we say that  $G$  is \_\_\_\_\_.
9. We are able to prove that every language accepted by an NTM is accepted by a TM. In this proof, we construct a TM that emulates the NTM using a bitstring which we call a \_\_\_\_\_ string.

Other

1. [30 points] Consider the NFA whose transition diagram is drawn below, where the input alphabet is  $\{a, b, c\}$ . Draw the transition diagram of an equivalent minimal DFA. Show your steps.

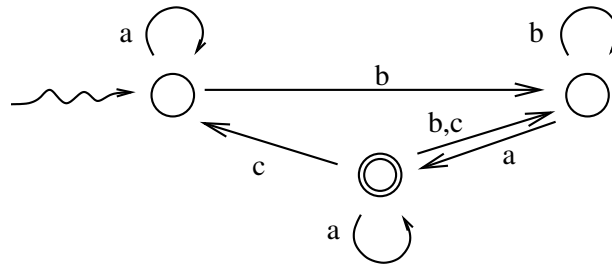


Figure 1: Find a minimal DFA equivalent to this NFA