Completing the Lecture of November 30, 2016

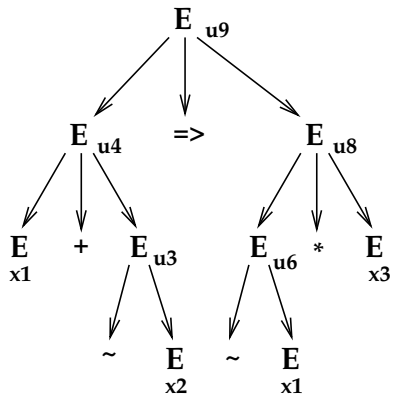
The notation I used during this semester (Fall 2016) differs slightly from the notation in the $\mathcal{NP}$ document, but not so much that you should be confused by it.

The problem was to show a reduction of SAT to 3SAT. In the $\mathcal{NP}$ document I refer to 3SAT $L_{\text{3cnf-sat}}$. That reduction R is completely explained in the $\mathcal{NP}$ document. However, I did not finish with the one example I had on the board on November 30.

That was to give $R(e)$, where:

$$e \equiv (x1+ \sim x2) \Rightarrow \sim x1 * x3$$

The parse tree for e, using an ambiguous version of context-free grammar given in the $\mathcal{NP}$ handout:



Our next step is to translate the parse tree into the conjunction of clauses:

$$(u4 = x1 + u3) * (u3 = \sim x2) * (u6 = \sim x1) * (u8 = u6 * x3) * (u9 = u4 \Rightarrow u8) * u9$$

Finally, we translate into 3-CNF form by replacing each of those clauses by the conjunction of CNF clauses of at most three terms:
$(u4+ \sim x1) * (u4+ \sim u3) * (\sim u4 + x1 + u3) *$
$(u3 + x2) * (\sim u3+ \sim x2) *$
$(u6 + x1) * (\sim u6+ \sim x1) *$
$(u8+ \sim u6+ \sim x3) * (\sim u8 + u6) * (\sim u8 + x3) *$
$(u9 + u4) * (u9+ \sim u8) * (\sim u9+ \sim u4 + u8) *$
$u9$

We can then "pad" each clause of length less than three by duplicating terms, so that each clause has exactly three terms. For example, we can replace $(u4+ \sim x1)$ with $(u4 + u4+ \sim x1)$, and $u9$ with $(u9 + u9 + u9)$.