## Instantaneous Description (id)

An **id** is a string, the concatenation of the following three strings.

1. The contents of the tape before the current cell, possibly the empty string.
2. The current state.
3. The contents of the current cell, one symbol.
4. The contents of the tape after the current cell but with trailing blanks deleted; possibly the empty string.

We denote a blank by "$\sqcup$." A state $q_i$ for an integer $i$, and $q_1$ is the start state. Each transition are labeled as $a \to b, M$, where $a$ is the symbol at the current cell, $b$ is symbol written at the current cell, and $M$ is the movement of the read-write head, either $L$, $R$ or $S$, for left, right, and stay. If $a = b$, we write $a \to M$ for short. The input string $w$ is initially in the first $|w|$ cells, and the leftmost cell is the initial current cell. Thus, the initial **id** is $q_1 w$.

## Example

Figure 1 illustrates a TM $M$ which decides the language $\left\{0^{2^k}\right\}$. The input alphabet is $\{0\}$ and the tape alphabet is $\{0, x, \sqcup\}$. If the input is accepted, $M$ halts in state $q_6$, otherwise it halts in state $q_7$.

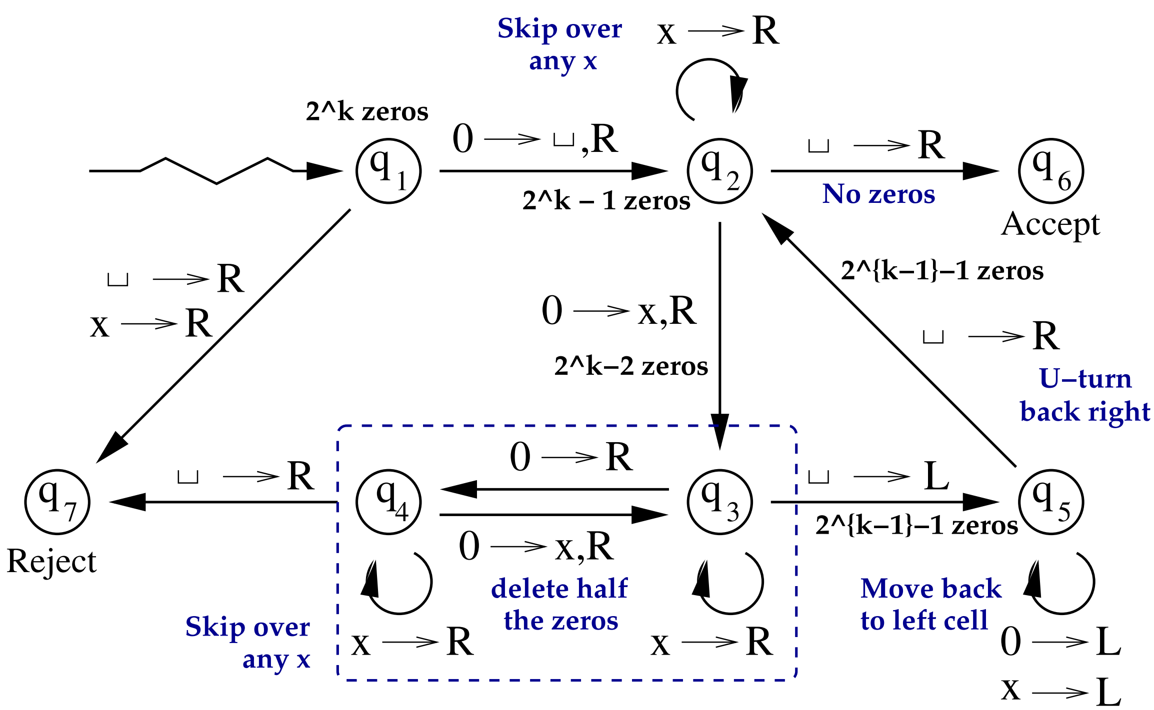


**Figure 1: State Diagram of $M$, a Turing Machine which Decides $\left\{0^{2^k}\right\}$**

## Explanation

At each state during an accepting comutation, that is, if the input is $0^{2^k}$, the number of zeros in the string is indicated in the diagram. The first zero is removed from $q_1$ to $q_2$. If there is none, the input is rejected. A zero is removed by replacing it by $x$.

An accepting computation iterates a loop from $q_2$ back to $q_2$, If there are $m$ zeros at $q_2$, and $m$ is odd, the back and forth movement between $q_3$ and $q_4$ deletes $\frac{m+1}{2}$ zeros. If $m = 2^k$, after $k$ iterations of the loop, the computation reaches $q_2$ with no zeros on the tape, and the next step is to accept by moving to $q_6$. On the other had, if the initial number of zeros is not a power of 2, eventually there will be no zeros on the tape at $q_4$, and the next state is $q_7$, the reject state.

For example: if the input is $0^{16}$, there will be 15 zeros at $q_2$, then later 7 zeros at $q_2$, then 3, then 1, then 0, followed by acceptance. On the other hand, if the the input is $0^{12}$, there will at first be 11 zeros on the tape at $q_2$, then 5 the next time, then 2 the next time, leading to the current cell being blank at $q_4$, followed by rejection. We give computations of $M$ with input $0^n$ for values of $n$ from 1 to 6.

$q_1 0$
$\sqcup q_2 \sqcup$
$\sqcup\sqcup q_6 \sqcup$
Accept

$q_1 00$
$\sqcup q_2 0$
$\sqcup x q_3 \sqcup$
$\sqcup q_5 x$
$q_5 \sqcup x$
$\sqcup q_2 x$
$\sqcup x q_2 \sqcup$
$\sqcup x \sqcup q_6 \sqcup$
Accept

$q_1 000$
$\sqcup q_2 00$
$\sqcup x q_3 0$
$\sqcup x 0 q_4 \sqcup$
$\sqcup x 0 \sqcup q_7 \sqcup$
Reject

$q_1 0000$
$\sqcup q_2 000$
$\sqcup x q_3 00$
$\sqcup x 0 q_4 0$
$\sqcup x 0 x q_3 \sqcup$
$\sqcup x 0 q_5 x$
$\sqcup x q_5 0 x$
$\sqcup q_5 x 0 x$
$q_5 \sqcup x 0 x$

$\sqcup q_2 x0x$
$\sqcup x q_2 0x$
$\sqcup xx q_3 x$
$\sqcup xxx q_3 \sqcup$
$\sqcup xx q_5 x$
$\sqcup x q_5 xx$
$\sqcup q_5 xxx$
$q_5 \sqcup xxx$
$\sqcup q_2 xxx$
$\sqcup x q_2 xx$
$\sqcup xx q_2 x$
$\sqcup xxx q_2 \sqcup$
$\sqcup xxx \sqcup q_6 \sqcup$
Accept

$q_1 00000$
$\sqcup q_2 0000$
$\sqcup x q_3 000$
$\sqcup x0 q_4 00$
$\sqcup x0x q_3 0$
$\sqcup x0x0 q_4 \sqcup$
$\sqcup x0x0 \sqcup q_7 \sqcup$
Reject

$q_1 000000$
$\sqcup q_2 00000$
$\sqcup x q_3 0000$
$\sqcup x0 q_4 000$
$\sqcup x0x q_3 00$
$\sqcup x0x0 q_4 0$
$\sqcup x0x0x q_3 \sqcup$
$\sqcup x0x0 q_5 x$
$\sqcup x0x q_5 0x$
$\sqcup x0 q_5 x0x$
$\sqcup x q_5 0x0x$
$\sqcup q_5 x0x0x$
$q_5 \sqcup x0x0x$
$\sqcup q_2 x0x0x$
$\sqcup x q_2 0x0x$
$\sqcup xx q_3 x0x$
$\sqcup xxx q_3 0x$
$\sqcup xxx0 q_4 x$
$\sqcup xxx0x q_4 \sqcup$
$\sqcup xxx0x \sqcup q_7 \sqcup$
Reject