Instantaneous Description (id)

An id is a string, the concatenation of the following three strings.

1. The contents of the tape before the current cell, possibly the empty string.
2. The current state.
3. The contents of the current cell, one symbol.
4. The contents of the tape after the current cell but with trailing blanks deleted; possibly the empty string.

We denote a blank by "⊔." A state $q_i$ for an integer $i$, and $q_1$ is the start state. Each transition are labeled as $a \rightarrow b, M$, where $a$ is the symbol at the current cell, $b$ is symbol written at the current cell, and $M$ is the movement of the read-write head, either $L$, $R$ or $S$, for left, right, and stay. If $a = b$, we write $a \rightarrow M$ for short. The input string $w$ is initially in the first $|w|$ cells, and the leftmost cell is the initial current cell. Thus, the initial id is $q_1w$.

Example

Figure 1 illustrates a TM $M$ which decides the language $\{0^{2^k}\}$. The input alphabet is $\{0\}$ and the tape alphabet is $\{0, x, ⊔\}$. If the input is accepted, $M$ halts in state $q_6$, otherwise it halts in state $q_7$. 

![State Diagram of M, a Turing Machine which Decides $\{0^{2^k}\}$](image-url)
Explanation

At each state during an accepting computation, that is, if the input is \(0^{2^k}\), the number of zeros in the string is indicated in the diagram. The first zero is removed from \(q_1\) to \(q_2\). If there is none, the input is rejected. A zero is removed by replacing it by \(x\).

An accepting computation iterates a loop from \(q_2\) back to \(q_2\). If there are \(m\) zeros at \(q_2\), and \(m\) is odd, the back and forth movement between \(q_3\) and \(q_4\) deletes \(\frac{m+1}{2}\) zeros. If \(m = 2^k\), after \(k\) iterations of the loop, the computation reaches \(q_2\) with no zeros on the tape, and the next step is to accept by moving to \(q_6\). On the other hand, if the initial number of zeros is not a power of 2, eventually there will be no zeros on the tape at \(q_4\), and the next state is \(q_7\), the reject state.

For example: if the input is \(0^{16}\), there will be 15 zeros at \(q_2\), then later 7 zeros at \(q_2\), then 3, then 1, then 0, followed by acceptance. On the other hand, if the the input is \(0^{12}\), there will at first be 11 zeros on the tape at \(q_2\), then 5 the next time, then 2 the next time, leading to the current cell being blank at \(q_4\), followed by rejection. We give computations of \(M\) with input \(0^n\) for values of \(n\) from 1 to 6.
\[\begin{align*}
\text{Accept} & \text{ } q_100000 \\
& \text{ } q_20000 \\
& \text{ } q_30000 \\
& \text{ } x0q_4000 \\
& \text{ } x0q_50 \\
& \text{ } x0x0q_50 \\
& \text{ } x0x0q_530 \\
& \text{ } x0x0q_540 \\
& \text{ } x0x0x0q_530 \\
& \text{ } x0x0x0q_560 \\
& \text{ } x0x0x0q_570 \\
\text{Reject} & \text{ } q_1000000 \\
& \text{ } q_200000 \\
& \text{ } q_300000 \\
& \text{ } x0q_40000 \\
& \text{ } x0q_5000 \\
& \text{ } x0x0q_5300 \\
& \text{ } x0x0q_5600 \\
& \text{ } x0x0x0q_5330 \\
& \text{ } x0x0x0q_5603 \\
& \text{ } x0x0x0q_5604 \\
& \text{ } x0x0x0q_5605 \\
& \text{ } x0x0x0q_5606 \\
& \text{ } x0x0x0q_5607 \\
\end{align*}\]