University of Nevada, Las Vegas Computer Science 456/656 Spring 2019

Assignment 4: Due March 6, 2019

Name:____

Print this document and staple, along with any extra sheets you want graded. Fill in answers by hand, not by typing or by computer. Hand it to the graduate assistant at the beginning of class on March 6. You are permitted to work in groups, get help from others, read books, and use the internet. But the handwriting on this document must be your own.

Recall that a language L is called *decidable* there is some machine M such that if M is given any string w as input, M will give the output "1" if $w \in L$, "0" if $w \notin L$. Decidable languages are also called *recursive.*

A function is said to be computable, or recursive, if it is computed by some machine. There are uncomputable functions, such as the Busy Beaver function:

`http://googology.wikia.com/wiki/Busy_beaver_function`

In fact, there are more uncomputable functions than computable functions.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (a) ______ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

   (b) ______ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   (c) ______ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (d) ______ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$.

   (e) ______ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (f) ______ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

   (g) ______ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (h) ______ The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.

   (i) ______ The intersection of any regular languages with any context-free language is context-free.

   (j) ______ If L is a context-free language over an alphabet with just one symbol, then L is regular.

   (k) ______ $\mathcal{P} = \mathcal{NP}$.

   (l) ______ There is a deterministic parser for any context-free grammar.

   (m) ______ The Boolean Circuit Problem is in $\mathcal{P}$.

(n) ______ The Boolean Circuit Problem is in $\mathcal{NC}$.

(o) ______ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

(p) ______ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

(q) ______ The language consisting of all strings over $\{a, b\}$ which have more a's than b's is context-free.

(r) ______ Every context-free language is in $\mathcal{P}$.

(s) ______ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

(t) ______ The problem of whether two given context-free grammars generate the same language is decidable.

(u) ______ The problem of whether a given string is generated by a given context-free grammar is decidable.

(v) ______ Every bounded function is recursive.

(w) ______ There exists a mathematical proposition that can be neither proved nor disproved.

(x) ______ There is a non-recursive function which grows faster than any recursive function.

(y) ______ There exists a machine[1] that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(z) ______ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

[1] As always in automata theory, "machine" means abstract machine, a mathematical object whose memory and running time are **not** constrained by the size and lifetime of the known (or unknown) universe, or any other physical laws. If we want to discuss the kind of machine that exists (or could exist) physically, we call it a "physical machine."

2. The language $L = \left\{a^i b^j c^k : i = j \text{ or } j = k\right\}$ is generated by the CF grammar G with start state S given below.

    1. $S \to XC$
    2. $X \to \varepsilon$
    3. $X \to aXb$
    4. $C \to \varepsilon$
    5. $C \to cC$
    6. $S \to AY$
    7. $A \to \varepsilon$
    8. $A \to aA$
    9. $Y \to \varepsilon$
    10. $Y \to bYc$

    Design a PDA which makes use the grammar G and accepts L. Walk through the steps of the PDA for the input string *abbcc*.