University of Nevada, Las Vegas Computer Science 456/656 Spring 2019

Assignment 6: Due April 8, 2019

Name:_____

Print this document and staple, along with any extra sheets you want graded. Fill in answers by hand, not by typing or by computer. Hand it to the graduate assistant at the beginning of class on April 1. You are permitted to work in groups, get help from others, read books, and use the internet. But the handwriting on this document must be your own.

1. Consider the following well-known complexity classes.

$$\mathcal{NC} \subseteq \mathcal{P} - \text{TIME} \subseteq \mathcal{NP} - \text{TIME} \subseteq \mathcal{P} - \text{SPACE} \subseteq \mathbf{EXP} - \text{TIME} \subseteq \mathbf{EXP} - \text{SPACE}$$

   We do not know which ones of these classes are equal. For the next two questions, assume that they're all different. The *mover's problem* is, given a room with a door and pieces of furniture of various shapes and sizes, can the furniture be moved into the room through the door?

   The *crane operator's problem* is, given a room and pieces of furniture of various shapes and sizes, can the furniture be placed into the room after the roof is removed?

   For both furniture problems, we assume that no piece of furniture can ever be fully or partially on top of another.

   (a) Which of the above complexity classes is the smallest class which is known to contain the mover's problem?

   (b) Which of the above complexity classes is the smallest class which is known to contain the crane operator's problem?

   (c) Which of the above complexity classes is the smallest class which is known to contain the context-free grammar membership problem?

   (d) Which of the above complexity classes is the smallest class which is known to contain the circuit valuation problem, which is the problem of determining the output of a Boolean circuit with given inputs?

   (e) *Generalized checkers* is the game of checkers played on an $n \times n$ board. (The standard game uses an 8×8 board.) Which of the above complexity classes is the smallest class which is known to contain the problem of determining whether the first player to move, from a given configuration, can win?

2. Write a proof of Theorem 1 below.

Theorem 1 *If a language L is recursive, there is a machine which enumerates L in canonical order.*

3. Write a proof of Theorem 2 below.

Theorem 2 *If there is a machine which enumerates a language L in canonical order, then L is recursive,*

4. Starting with the knowledge that the Knapsack problem is $\mathcal{NP}$-complete, prove that the Parttion problem is $\mathcal{NP}$-complete.