

Computer Science 456/656 Spring 2019 Practice Examination April 15, 2019

The entire examination is 640 points. The actual exam will be shorter.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.
 - i ----- Every subset of a regular language is regular.
 - ii ----- Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m, n \geq 0$. Then L is a regular language.
 - iii ----- The complement of every regular language is regular.
 - iv ----- The Kleene closure of every context-free language is context-free.
 - v ----- If a language has an ambiguous context-free grammar, then it is not accepted by any deterministic push-down automaton.
 - vi ----- There is a PDA that accepts all valid C++ programs.
 - vii ----- The intersection of any two regular languages is regular.
 - viii ----- The intersection of any two context-free languages is context-free.
 - ix ----- The set of all base 7 numerals for positive integers n such that $n \% 3 = 2$ is regular.
 - x ----- Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n c^m$, where $m, n \geq 0$. Then L is a context-free language.
 - xi ----- Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a context-free language.
 - xii ----- The complement of every context-free language is context-free.
 - xiii ----- The union of any two context-free languages is context-free.
 - xiv ----- If a language has a context-free grammar, then it is accepted by some push-down automaton.
 - xv ----- Every context-free language has an unambiguous context-free grammar.
 - xvi ----- Every language that has an unambiguous context-free grammar is accepted by some DPDA.
 - xvii ----- Every deterministic machine is a non-deterministic machine.
 - xviii ----- The language consisting of all base 2 numerals for integer powers of 2 is regular.
 - xix ----- There is a DPDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.
 - xx ----- The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

- xxi _____ The problem of whether a given context-free grammar generates all strings is decidable.
- xxii _____ The language $\{a^i b^j c^k \mid j \geq i + k\}$ is context-free.
- xxiii _____ If a language L is undecidable, there is no machine that enumerates L in canonical order.
- xxiv If \mathcal{L} is a class of languages, $\text{co-}\mathcal{L}$ is defined to be the class of all languages that are not in \mathcal{L} .
 _____ Let \mathcal{RE} be the class of all recursively enumerable languages. If L is in \mathcal{RE} and also L is in $\text{co-}\mathcal{RE}$, then L must be decidable.
- xxv _____ The context-free grammar equivalence problem is in the class \mathcal{RE} .
- xxvi _____ The context-free grammar equivalence problem is in the class $\text{co-}\mathcal{RE}$.
- xxvii _____ Every bounded function is recursive.
- xxviii _____ If P is a mathematical proposition that can be stated using n binary bits, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.
- xxix _____ The complement of every recursive language is recursive.
- xxx _____ The complement of every recursively enumerable language is recursively enumerable.
- xxxi _____ Every language which is generated by an general grammar is recursively enumerable.
- xxxii _____ The context-free grammar equivalence problem is undecidable.
- xxxiii _____ Given any context-free grammar G and any string $w \in L(G)$, there is always a unique leftmost derivation of w using G .
- xxxiv _____ For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.
- xxxv _____ Using multi-processors and other advanced technology, it is possible to design a machine which decides the halting problem.
- xxxvi _____ The question of whether two regular expressions are equivalent is \mathcal{NP} -complete.
- xxxvii _____ The intersection of any context-free language with any regular language is context-free.
- xxxviii _____ The halting problem is recursively enumerable.
- xxxix _____ The complement of every context-free language is context-free.
- xl _____ No language which has an ambiguous context-free grammar can be accepted by a DPDA.
- xli _____ The union of any two context-free languages is context-free.
- xl ii _____ The question of whether a given Turing Machine halts with empty input is decidable.
- xl iii _____ The class of languages accepted by non-deterministic Turing Machines is the same as the class of languages accepted by Turing Machines.

- xliv ----- Let $F(0) = 1$, and let $F(n) = 2^{F(n-1)}$ for $n > 0$. Then F is recursive.
 - xlvi ----- Every language which is accepted by some non-deterministic machine is accepted by some deterministic machine.
 - xlvi ----- The language of all regular expressions over the binary alphabet is a regular language.
 - xlvi ----- There is no computer program that decides whether any two C++ programs are equivalent.
 - xlvi ----- Every function that can be mathematically defined is recursive.
 - xlix ----- The language of all binary numerals for multiples of 23 is regular.
 - l ----- The language of all binary strings which are the binary numerals for prime numbers is context-free.
 - li ----- Every bounded function from integers to integers is Turing-computable. (We say that f is *bounded* if there is some B such that $|f(n)| \leq B$ for all n .)
 - lii ----- The language of all palindromes over $\{0, 1\}$ is inherently ambiguous.
 - liii ----- The language $\{a^i b^j c^k : i = j \text{ or } j = k\}$ is context-free, but is inherently ambiguous.
 - liv ----- Every context-free grammar can be parsed by some deterministic top-down parser.
 - lv ----- Every context-free grammar can be parsed by some non-deterministic top-down parser.
 - lvi ----- Commercially available parsers cannot use the LALR technique, since most modern programming languages are not context-free.
 - lvii ----- The diagonal language is \mathcal{RE} .
 - lviii ----- The diagonal language is $\text{co-}\mathcal{RE}$.
 - lix ----- The regular grammar membership problem is in \mathcal{NC} .
 - lx ----- The context-free grammar membership problem is in \mathcal{NC} .
 - lxi ----- There is a polynomial time reduction of SAT to 3-CNF-SAT.
 - lxii ----- There is a language which is neither \mathcal{RE} nor $\text{co-}\mathcal{RE}$.
2. Fill in the blanks.
- (a) [10 points] If there is an easy reduction from L_1 to L_2 , then ----- is at least as hard as -----.
 - (b) If a language is accepted by some Turing machine, it is ----- enumerable.
3. [30 points] State the Church-Turing thesis, and explain (in about 5 lines or less) why it is important.
4. [10 points] Consider the context-free grammar with start symbol S and productions as follows:
- 1 $S \rightarrow s$

$$2 \ S \rightarrow bLn$$

$$3 \ S \rightarrow wS$$

$$4 \ L \rightarrow \epsilon$$

$$5 \ L \rightarrow SL$$

Write a leftmost derivation of the string $bswsbwsnn$

5. [5 points] What class of machines accepts the class of context free languages?
6. [5 points] What class of machines accepts the class of recursively enumerable languages?
7. [10 points] What is the *canonical order* of a language?
8. [10 points] What does it mean to say that machines M_1 and M_2 are *equivalent*?
9. [10 points] What does it mean to say that a language L is *decidable*?
10. [10 points] Give a Chomsky Normal Form grammar for the Dyck language.
11. [10 points] Consider the context-free grammar G , with start symbol S and productions as follows:

$$1 \ S \rightarrow s$$

$$2 \ S \rightarrow bLn$$

$$3 \ S \rightarrow iS$$

$$4 \ S \rightarrow iSeS$$

$$5 \ L \rightarrow \epsilon$$

$$6 \ L \rightarrow LS$$

Prove that G is ambiguous by giving two different leftmost derivations for some string.

12. [20 points] Let G be the context-free grammar given below.

$$1 \ S \rightarrow a$$

$$2 \ S \rightarrow wS$$

$$3 \ S \rightarrow iS$$

$$4 \ S \rightarrow iSeS$$

Prove that G is ambiguous by writing two different **leftmost** derivations for the string $iwiwaea$. [If you simply show two different parse trees, you are not following instructions.]

13. [20 points] Let L be the language generated by the following context-free grammar.

$$1. \ S \rightarrow a$$

$$2. \ S \rightarrow wS$$

$$3. \ S \rightarrow iS$$

4. $S \rightarrow iSeS$

Write a Chomsky normal form grammar for L .

14. [30 points] Let $L = \{w \in \{a, b\}^* \mid \#_a(w) = 2\#_b(w)\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w . For example, $aaababaaabba \in L$, because that string has the twice as many a 's as b 's. Give a context-free grammar for L . Your grammar may be ambiguous.
15. [20 points] Let $\Sigma = \{0, 1\}$, the binary alphabet. We say a string w over Σ is *mostly positive* if $\#_1(w) > \#_0(w)$. Let L be the set of mostly positive strings over Σ .

Give a context-free grammar for L . **Very hard.**

16. [10 points] what is a **reduction** of a language L_1 to a language L_2 ?
17. [20 points] Consider the language L of all strings which would be acceptable as algebraic expressions involving variables and constants, where:

- Every variable name is either x , y , or z .
- Every constant is a natural number between 0 and 9
- The only operators are addition, subtraction, and multiplication.
- The symbol ' $-$ ' is used only for subtraction. There is no negation.
- There is no multiplication symbol. Multiplication is indicated by concatenating strings.
- In multiplication of a constant by anything else, the constant must come first, and there can be at most one constant factor in any term.
- Parentheses can be used.

Here are some strings in the language $x(y + 2z)$, $x - 1 - z$, $4(zx - 2y)(x + z(x - 1))$.

Give an unambiguous context-free grammar for L which is consistent with the usual semantics (as you learned in school) of such expressions.

18. (a) [10 points] State the pumping lemma for context-free languages accurately. If you have all the right words but in the wrong order, that means you truly do not understand the lemma, and you might get no partial credit at all.
- (b) [10 points] Let $L = \{a^i b^j c d^i \mid i, j \geq 0\}$. alphabet. What is the minimum pumping length of L ?
19. [10 points] Prove that a recursively enumerable language is accepted by some machine.
20. [10 points] Prove that a language is accepted by some machine if it is recursively enumerable.
21. [10 points] Prove that a language can be recursively enumerated in canonical order if it is recursive.
22. [10 points] Prove that a recursive language can be recursively enumerated in canonical order.
23. [20 points] Give a polynomial time reduction of the knapsack problem to the partition problem.
24. [20 points] Give a polynomial time reduction of 3-CNF-SAT to the independent set problem. You may use figures in your explanation.