**University of Nevada, Las Vegas**
**Computer Science 456/656 Spring 2020**

## Final Examination May 14, 2020

Final update: `Thu May 14 18:07:59 PDT 2020`

The entire examination is 405 points.

Name:____________________________________________________________

Print out this test and write your answers on the printout, then scan the pages and email to the GA, Pradip Maharjan,.with a time stamp of either May 14 PDt or May 15 PDT. If you are in a different time zone, be sure to adjust for that. For example, if you are in New York, your time stamp must be no later than 03:00 May 16 EDT, while if you are in Hawaii, your time stamp must be no later than May 15 21:00 HST.

If you need to attach extra pages, use white paper. If you are unable to print the test, you may write the answers on white paper. Mot lined paper, and not any other color. Please write large and dark enough that your answers are unambigurous Do **not** use lined paper, as it does not scan very well. Use only bright white paper. Make sure your name is on every page, in case pages get separated.

1. [5 points each] True or False. If the question is currently open, write "O" or "Open."

(i) ________ The complement of every regular language is regular.

(ii) ________ The complement of every context-free language is context-free.

(iii) ________ The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

(iv) ________ The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

(v) ________ The complement of any $\mathcal{P}$-SPACE language is $\mathcal{P}$-SPACE.

(vi) ________ The complement of every recursive language is recursive.

(vii) ________ The complement of every recursively enumerable language is recursively enumerable.

(viii) ________ Every language which is generated by a general grammar is recursively enumerable.

(ix) ________ The context-free membership problem is undecidable.

(x) ________ The factoring problem, where inputs are written in binary notation, is co-$\mathcal{NP}$.

(xi) ________ If $L_1$ reduces to $L_2$ in polynomial time, and if $L_2$ is $\mathcal{NP}$, and if $L_1$ is $\mathcal{NP}$-complete, then $L_2$ must be $\mathcal{NP}$-complete.

(xii) ________ Given any context-free grammar $G$ and any string $w \in L(G)$, there is always a unique leftmost derivation of $w$ using $G$.

(xiii) ________ For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

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(xiv) ________ The question of whether two regular expressions are equivalent is known to be $\mathcal{NP}$-complete.

(xv) ________ The halting problem is recursively enumerable.

(xvi) ________ The union of any two context-free languages is context-free.

(xvii) ________ The question of whether a given Turing Machine halts with empty input is decidable.

(xviii) ________ The class of languages accepted by non-deterministic finite automata is the same as the class of languages accepted by deterministic finite automata.

(xix) ________ The class of languages accepted by non-deterministic push-down automata is the same as the class of languages accepted by deterministic push-down automata.

(xx) ________ The class of languages accepted by non-deterministic Turing Machines is the same as the class of languages accepted by deterministic Turing Machines.

(xxi) ________ The intersection of any two context-free languages is context-free.

(xxii) ________ If $L_1$ reduces to $L_2$ in polynomial time, and if $L_2$ is $\mathcal{NP}$, then $L_1$ must be $\mathcal{NP}$.

(xxiii) ________ Let $\pi$ be the ratio of the circumference of a circle to its diameter. The problem of whether the $n^{\text{th}}$ digit of the decimal expansion of $\pi$ for a given $n$ is equal to a given digit is decidable.

(xxiv) ________ There cannot exist any computer program that can decide whether any two C++ programs are equivalent.

(xxv) ________ Every context-free language is in the class $\mathcal{P}$-TIME.

(xxvi) ________ Every regular language is in the class $\mathcal{NC}$

(xxvii) ________ The language of all binary numerals for multiples of 23 is regular.

(xxviii) ________ The language of all binary strings which are the binary numerals for prime numbers is context-free.

(xxix) ________ Every context-free grammar can be parsed by some non-deterministic top-down parser.

(xxx) ________ If anyone ever proves that $\mathcal{P} = \mathcal{NP}$, then all one-way encoding systems will be insecure.

(xxxi) ________ If a string $w$ is generated by a context-free grammer $G$, then $w$ has a unique leftmost derivation if and only if it has a unique rightmost derivation.

(xxxii) ________ A language $L$ is in $\mathcal{NP}$ if and only if there is a polynomial time reduction of $L$ to SAT.

(xxxiii) ________ A language $L$ is in $\mathcal{P}$-SPACE if and only if there is a polynomial time reduction of $L$ to some contexc-sensitive language.

(xxxiv) ________ A language $L$ is in recursively enumerable if and only if there is a recursive reduction of $L$ to the halting problem.

(xxxv) ________ A language $L$ is in $\mathcal{P}$-TIME if and only if there is an $\mathcal{NC}$ reduction of $L$ to Boolean satisfiability.

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2. [5 points] What class of machines accepts the class of context free languages?

3. [5 points] What class of machines accepts the class of recursively enumerable languages?

4. [20 points] Using the context-free grammar with start symbol $S$ and productions listed below, write two different leftmost derivations (not parse trees) of the string $iibwaanea$

$S \to a$
$S \to bLn$
$S \to wS$
$S \to iS$
$S \to iSeS$
$L \to \lambda$
$L \to SL$

5. [20 points] Draw an NFA with five states which accepts the language described by the regular expression $(a+b)^*a(a+b)(a+b)(a+b)$

6. [20 points] Draw a DFA which accepts the language $L$ over the alphabet $\{a, b, c\}$ consisting of all strings which contain either $aba$ or $caa$ as a substring. (My answer has six states.)
**Update: there is a 5 state solution.** To think about: What do my labels mean?

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7. [20 points] Find a context-free grammar which generates the language $L = \left\{a^i b^j c^k : i = j \text{ or } i = k\right\}$

8. [20 points] Draw a state diagram for a PDA that accepts the Dyck language. (For ease of grading, use $a$ and $b$ instead of "[" and "]")

9. [20 points] Draw the state diagram for a DFA that accepts the language described by the regular expression $(\mathtt{a}(\lambda\mathtt{+b+bb})\mathtt{a})^*$

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10. [20 points] Let $L$ be the language of all binary numerals for positive integers which are multiples of 4. Thus, for example, the binary numerals for 0, 4, 8, 12, 16, 20 ... are in $L$. We allow a binary numeral to have leading zeros; thus (for example) $0011100 \in L$, since it is a binary numeral for 28. Draw a DFA with four states which accepts $L$.

11. [20 points] Prove that every decidable language can be enumerated in canonical order by some machine.

12. [20 points] State the Church-Turing Thesis.

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13. [20 points] Find a $\mathcal{P}$-time reduction of the subset sum problem to the partion problem.

14. [20 points] Let $\Sigma$ be the Boolean alphabet. Here is a "proof" that every language $L$ over $\Sigma$ is decidable.

"For any $n \geq 0$, let $\Sigma^n$ be the set of strings over $\Sigma$ of length $n$, and let $L_n = L \cap \Sigma^n$. $L_n$ is finite, in fact, $|L_n| \leq 2^n$. Thus, $L_n$ is decidable. Let $\mathcal{P}$ be the following program:

Read a string $w \in \Sigma^*$.
    Let $n = |w|$
If $(w \in L_n)$        (Remember: $L_n$ is decidable)
    Write "yes" $(w \in L)$
else
    Write "no" $(w \notin L)$

$\mathcal{P}$ decides $L$. We conclude that every language is decidable."

But, since HALT is undecidable, this proof can't be right. What's wrong with it?