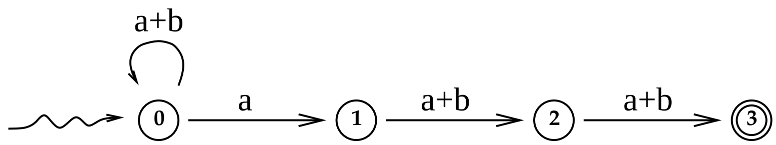
University of Nevada, Las Vegas Computer Science 456/656 Spring 2020

Answers to Assignment 2: Due Thursday February 13, 2020

1. Write a regular expression for the language consisting of all strings over $\{a, b\}$ which contain the substring aaa.

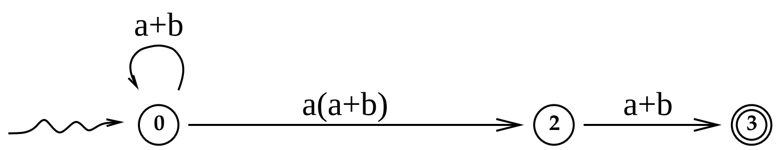
$$(a + b)^* aaa(a + b)^*$$

2. Use the method given on page 86 of the sixth edition of Linz, or on page 89 of the fifth edition, to find a regular expression equivalent to the following NFA.

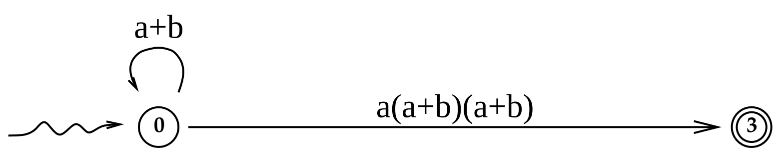


In our construction, to avoid clutter, we do not write any self-loop labeled λ, and we do not write any arc labeled $\emptyset$. This omissions is justified by the identities $\lambda x = x\lambda = x$, $\emptyset x = x\emptyset = \emptyset$, where x is any regular expression.

Eliminate ①:

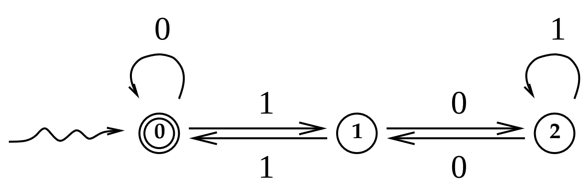


Eliminate ②:

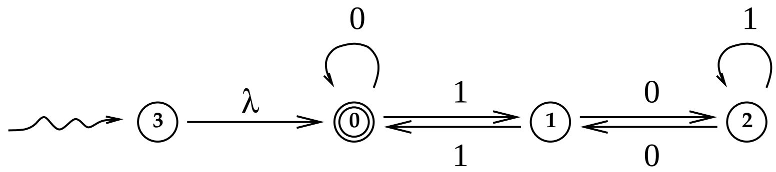


The regular expression is $(a + b)^* a(a + b)(a + b)$.

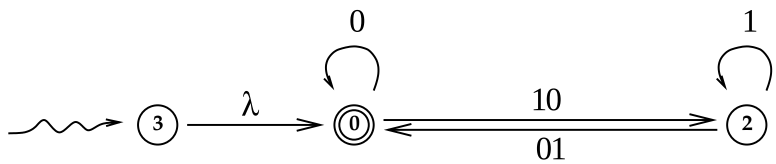
3. The following DFA accepts the language consisting of all binary numerals for positive multiples of three, where a leading 0 is allowed. Use the method given on page 86 of the sixth edition of Linz, or on page 89 of the fifth edition, to find an equivalent regular expression.



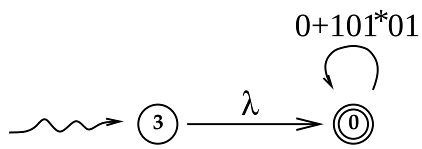
Since the start state is final, introduce a dummy start state:



Eliminate ①:



Eliminate ②:



The regular expression is $(0 + 101^*01)^*$.

4. (a) State the pumping lemma for regular languages.

For any regular language L there exists a positive integer p, the pumping length of L, such that for any $w \in L$ whose length is at least p there exist tring x, y, z such that:

i. $w = xyz$,
ii. $|y| \geq 1$,
iii. $|xy| \leq p$,
iv. for any integer $i > 0$, $xy^iz \in L$.

(b) Use the pumping lemma to prove that the language $L = \{a^nb^n : n \geq 0\}$ is not regular.

Assume L is regular. Let p be a pumping length of L. Let $w = a^pb^p$. Then there exist strings x, y, z such that

i. $w = xyz$,
ii. $|y| \geq 1$,
iii. $|xy| \leq p$,
iv. for any integer $i > 0$, $xy^iz \in L$.

By (ii) $|y| = k$ for some $k > 0$. By (i) and (iii) xy contains no b, hence y contains to b. By (iv) $xy^0z = a^{p-k}b^p \in L$, contradition because $p - k \neq p$. Thus L is not regular.

5. Work problem 9(a) on page 138 of the sixth edition, which is problem 7(a) on page 137 of the fifth edition.

   $S \to AAAT$
   $A \to a \,|\, \lambda$
   $T \to aTb \,|\, Tb \,|\, \lambda$

6. Work problem 9(c) on page 138 of the sixth edition, which is problem 7(c) on page 137 of the fifth edition.

   $S \to S_1 \,|\, S_2$
   $S_1 \to aS_1 \,|\, aT_1$
   $T_1 \to aaT_1b \,|\, \lambda$
   $S_2 \to S_2b \,|\, T_1b$
   $T_2 \to aaT_2b \,|\, \lambda$

7. Work problem 24 on page 140 of the sixth edition, which is problem 22 on page 139 of the fifth edition.

   The following unambiguous grammar generates the language. There are also ambiguous grammars for the same language. $S \to (S)S \,|\, [S]S \,|\, \lambda$

8. Work problem 25 on page 140 of the sixth edition, which is problem 23 on page 139 of the fifth edition. Let R be the language of all regular expressions over $\{a, b\}$.

   The alphabet for R is $\Sigma = \{a, b, \lambda, \emptyset, +, ^*, (,)\}$ There is a simple ambiguous context-free grammar for R:

   $S \to SS \,|\, S + S \,|\, (S) \,|\, S^* \,|\, (S) \,|\, a \,|\, b \,|\, \lambda \,|\, \emptyset$

   On the other hand, we might prefer an unambigous context-free grammar. We use the usual method of constructing a grammar for an algebraic language, using variables E, T, and F, where E is the start symbol.

   $E \to E + T \,|\, T$
   $T \to TF \,|\, F$
   $F \to F^* \,|\, (E) \,|\, a \,|\, b \,|\, \lambda \,|\, \emptyset$