## Computer Science 456/656 Spring 2020

## Practice for First Examination February 27, 2020

The entire examination is 420 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (a) ________ Every subset of a regular language is regular.

   (b) ________ The Dyck language is regular.

   (c) ________ If a language $L$ is generated by some context-free grammar, then $L$ is accepted by some PDA.

   (d) ________ If $L$ is a language accepted by some PDA, then $L$ is generated by some context-free grammar.

   (e) ________ The Kleene closure of every context-free language is context-free.

   (f) ________ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   (g) ________ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   (h) ________ There is a PDA that accepts the language consisting of all C++ programs.

   (i) ________ Let $L$ be the language over $\Sigma = \{a, b, c\}$ consisting of all strings of the form $a^n b^n c^n$, where $n \geq 0$. Then $L$ is a context-free language.

   (j) ________ Let $L$ be the language over $\Sigma = \{a, b, c, d\}$ consisting of all strings of the form $a^n b^m c^p d^q$, where $0 \leq n \leq q$ and $0 \leq m \leq p$. Then $L$ is a context-free language.

   (k) ________ The intersection of any two context-free languages is context-free.

   (l) ________ The union of any two context-free languages is context-free.

   (m) ________ The language $\{a^m b c^n : 0 \leq m \leq n\}$ is accepted by some DPDA.

   (n) ________ The membership problem for context-free languages is decidable.

   (o) ________ The equivalence problem for context-free grammars is decidable.

   (p) ________ Every DFA is an NFA.

   (q) ________ Let $L$ be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, for any $m$ and $n$. Then $L$ is a regular language.

   (r) ________ Let $L$ be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then $L$ is a regular language.

   (s) ________ Every regular language is context-free.

   (t) ________ The Kleene closure of every regular language is regular.

   (u) ________ The language consisting of all hexadecimal numerals for positive integers $n$ such that $n \% 13 = 7$ is regular.

   (v) ________ The complement of every regular language is regular.

   (w) ________ The union of any two regular languages is regular.

(x) ________ Every NFA is a DFA.

(y) ________ The intersection of any two regular languages is regular.

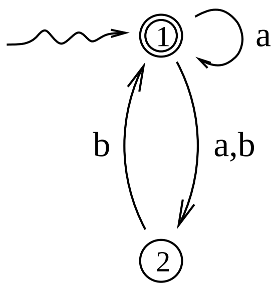
(z) ______ There exists a mathematical proposition that is true, but where no proof of the proposition can exist.

2. [20 points] Let $L$ be the language consisting of all strings over the binary alphabet whose last three symbols are '`010`.' Draw an NFA with four states which accepts $L$.

3. [20 points] Describe the language $L$ generated by the following context-free grammar where $\{a, b\}$ is the set of terminals, $\{S\}$ is the set of variables, $S$ is the start symbol, and the productions are as follows:

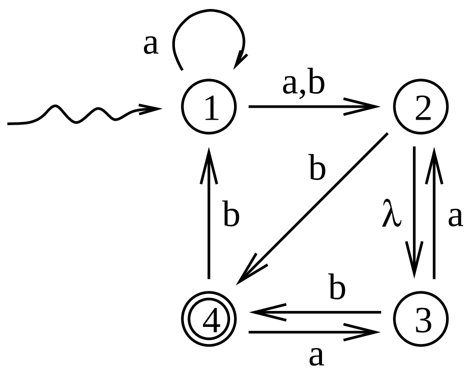
   1. $S \to aSb$
   2. $S \to aS$
   3. $S \to \varepsilon$

4. [20 points] Write a regular expression for the language accepted by the NFA shown below.




5. [20 points] Let $L$ be the language consisting of all strings over $\{a, b\}$ which do not contain the substring $aab$. Write a regular expression for $L$ and draw a minimal DFA which accepts $L$. (Hint: 3 states.)

6. [40 points] Draw a state diagram for a minimal DFA equivalent to the NFA shown below. Partial credit if you get the first steps correct.



7. [5 points] The ______________ algorithm decides whether a given string is a member of a given context-free language.

8. [5 points] ______________________________________________________________________________________ has an unambiguous context-free grammar, but is not accepted by any DPDA.

9. [30 points] State the pumping lemma for regular languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

10. [20 points] Let $G$ be the context-free grammar given below.

$S \to a$

$S \to wS$

$S \to iS$

$S \to iSeS$

(a) Prove that $G$ is ambiguous by writing two different **rightmost** derivations for the string *iwiaea*. [If you simply show two different parse trees, you are not following instructions.]

(b) Give a CNF grammar equivalent to $G$.

11. [30 points] Draw the state diagram for a PDA that accepts $\{a^n b^n : n \geq 0\}$.

12. [30 points] The following context-free grammar $G$ is ambiguous. Give an equivalent unambiguous grammar. (Hint: the "standard" solution to this problem is a grammar that we have discussed in class that has three variables.)

1. $E \rightarrow E + E$
2. $E \rightarrow E - E$
3. $E \rightarrow E * E$
4. $E \rightarrow - E$
5. $E \rightarrow ( E )$
6. $E \rightarrow x$
7. $E \rightarrow y$
8. $E \rightarrow z$

13. [20 points] Let $L$ be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^n b^n$ for $n \geq 1$. Give a Chomsky Normal Form grammar for $L$.

14. [30 points] Let $L$ be the language generated by the Chomsky Normal Form (CNF) grammar given below.

(a) $S \to a$

(b) $E \to a$

(c) $S \to LA$

(d) $E \to LA$

(e) $L \to ($

(f) $A \to ER$

(g) $R \to )$

(h) $S \to PE$

(i) $E \to PE$

(j) $S \to EE$

(k) $E \to EE$

(l) $P \to EQ$

(m) $Q \to +$

Use the CYK algorithm to prove that the string $a(a + a)$ is a member of $L$. Use the figure below for your work.