Computer Science 456/656 Spring 2020

Answers to Practice for First Examination February 27, 2020

The entire examination is 420 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (a) **F** Every subset of a regular language is regular.

   (b) **F** The Dyck language is regular.

   (c) **T** If a language L is generated by some context-free grammar, then L is accepted by some PDA.

   (d) **T** If L is a language accepted by some PDA, then L is generated by some context-free grammar.

   (e) **T** The Kleene closure of every context-free language is context-free.

   (f) **F** If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   (g) **F** If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   (h) **F** There is a PDA that accepts the language consisting of all C++ programs.

   (i) **F** Let L be the language over $\Sigma = \{a, b, c\}$ consisting of all strings of the form $a^n b^n c^n$, where $n \geq 0$. Then L is a context-free language.

   (j) **T** Let L be the language over $\Sigma = \{a, b, c, d\}$ consisting of all strings of the form $a^n b^m c^p d^q$, where $0 \leq n \leq q$ and $0 \leq m \leq p$. Then L is a context-free language.

   (k) **F** The intersection of any two context-free languages is context-free.

   (l) **T** The union of any two context-free languages is context-free.

   (m) **T** The language $\{a^m b c^n : 0 \leq m \leq n\}$ is accepted by some DPDA.

   (n) **T** The membership problem for context-free languages is decidable.

   (o) **F** The equivalence problem for context-free grammars is decidable.

   (p) **T** Every DFA is an NFA.

   (q) **T** Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, for any m and n. Then L is a regular language.

   (r) **F** Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a regular language.

   (s) **T** Every regular language is context-free.

   (t) **T** The Kleene closure of every regular language is regular.

   (u) **T** The language consisting of all hexadecimal numerals for positive integers n such that $n \% 13 = 7$ is regular.

   (v) **T** The complement of every regular language is regular.

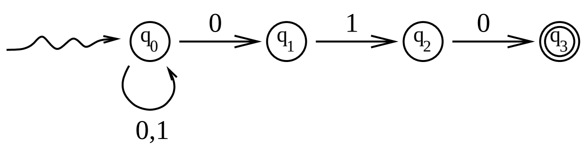
   (w) **T** The union of any two regular languages is regular.

(x) **F** Every NFA is a DFA.

(y) **T** The intersection of any two regular languages is regular.

(z) **T** There exists a mathematical proposition that is true, but where no proof of the proposition can exist.

2. [20 points] Let L be the language consisting of all strings over the binary alphabet whose last three symbols are '`010`.' Draw an NFA with four states which accepts L.

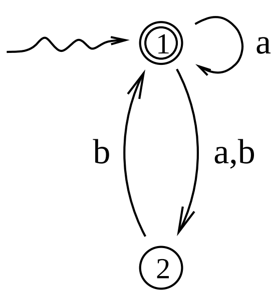


3. [20 points] Describe the language L generated by the following context-free grammar where $\{a, b\}$ is the set of terminals, $\{S\}$ is the set of variables, S is the start symbol, and the productions are as follows:

   1. $S \to aSb$
   2. $S \to aS$
   3. $S \to \lambda$

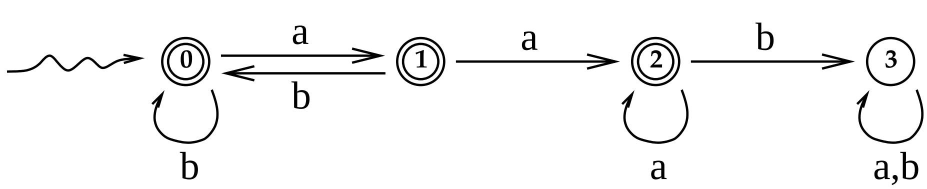
   $\{a^n b^m \,:\, 0 \le m \le n\}$

4. [20 points] Write a regular expression for the language accepted by the NFA shown below.



$(\texttt{a+ab+bb})^*$

5. [20 points] Let L be the language consisting of all strings over $\{a, b\}$ which do not contain the substring aab. Write a regular expression for L and draw a minimal DFA which accepts L. (Hint: 3 states, if you don't count state 3, which is the dead state.)

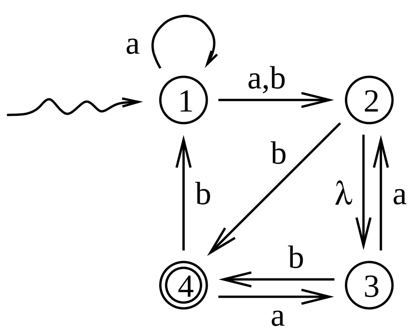


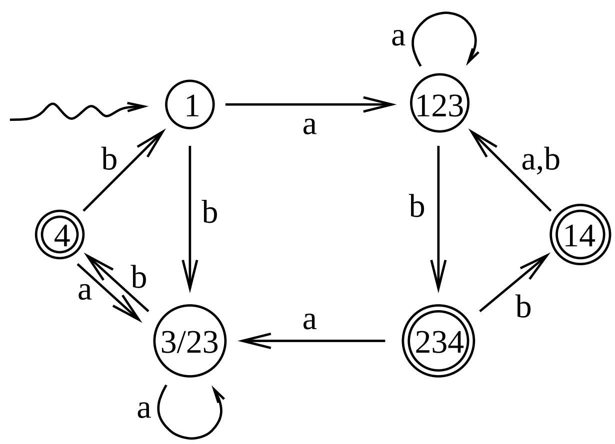
$(\texttt{ab} + \texttt{b})^*\texttt{a}^*$

7. [5 points] The CYK algorithm decides whether a given string is a member of a given context-free language.

6. [40 points] Draw a state diagram for a minimal DFA equivalent to the NFA shown below. Partial credit if you get the first steps correct.

	a	b
1	123	23
3	23	4
4	3	1
23	23	4
123	123	234
234	23	14
14	123	123





8. [5 points] ____ has an unambiguous context-free grammar, but is not accepted by any DPDA.

There are many correct answers. The easiest answer that I know is the language of all palindromes over an alphabet of size 2.

9. [30 points] State the pumping lemma for regular languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

For any regular languge L there is a positive integer p such that for any $w \in L$ of length at least p, there exist strings x, y, z, such that the following four conditions hold:

1. $w = xyz$
2. $y \neq \lambda$
3. $|xy| \leq p$
4. for any integer $i \geq 0$, $xy^iz \in L$.

10. [20 points] Let G be the context-free grammar given below.

$S \to a$

$S \to wS$

$S \to iS$

$S \to iSeS$

(a) Prove that G is ambiguous by writing two different **rightmost** derivations for the string $iwiaea$. [If you simply show two different parse trees, you are not following instructions.]

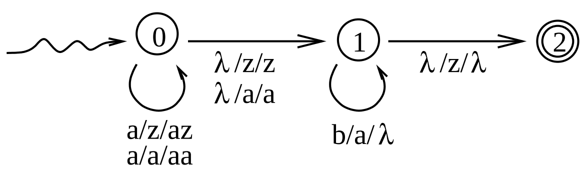
$$S \Rightarrow iSeS \Rightarrow iSea \Rightarrow iwSea \Rightarrow iwiSea \Rightarrow iwiaea$$

$$S \Rightarrow iS \Rightarrow iwS \Rightarrow iwiSeS \Rightarrow iwiSea \Rightarrow iwiaea$$

(b) Give a CNF grammar equivalent to G.

$S \to IS$
$S \to WS$
$S \to XY$
$X \to IS$
$Y \to ES$
$S \to a$
$W \to w$
$I \to i$
$E \to e$

11. [30 points] Draw the state diagram for a PDA that accepts $\{a^n b^n : n \geq 0\}$.



12. [30 points] The following context-free grammar G is ambiguous. Give an equivalent unambiguous grammar. (Hint: the "standard" solution to this problem is a grammar that we have discussed in class that has three variables.)

$E \to E + E$	$E \to E + T$
$E \to E - E$	$E \to E - T$
$E \to E * E$	$E \to T$
$E \to - E$	$T \to T * F$
$E \to (E)$	$T \to F$
$E \to x$	$F \to - F$
$E \to y$	$F \to (E)$
$E \to z$	$F \to x$
	$F \to y$
	$F \to z$

13. [20 points] Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^n b^n$ for $n \geq 1$. Give a Chomsky Normal Form grammar for L.

$S \to AB$
$S \to AT$
$T \to SB$
$A \to a$
$B \to b$

14. [30 points] Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

(a) $S \to a$

(b) $E \to a$

(c) $S \to LA$

(d) $E \to LA$

(e) $L \to ($

(f) $A \to ER$

(g) $R \to)$

(h) $S \to PE$

(i) $E \to PE$

(j) $S \to EE$

(k) $E \to EE$

(l) $P \to EQ$

(m) $Q \to +$

Use the CYK algorithm to prove that the string $a(a + a)$ is a member of L. Use the figure below for your work.