University of Nevada, Las Vegas Computer Science 456/656 Spring 2020

Practice Examination for April 30, 2020

Updated Fri Apr 24 13:55:39 PDT 2020

The entire practice examination is 480 points.

The current closure order extends to April 30. Thus the exam will be take-home.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (ii) _____ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

   (iii) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (iv) Deleted.

   (v) _____ The intersection of any three regular languages is context-free.

   (vi) _____ If L is a context-free language over an alphabet with just one symbol, then L is regular.

   (vii) _____ There is a deterministic parser for any context-free grammar.

   (viii) _____ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

   (ix) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (x) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (xi) _____ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

   (xii) _____ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

   (xiii) _____ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is in the class $\mathcal{P}$-TIME.

   (xiv) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (xv) _____ Every undecidable problem is $\mathcal{NP}$-complete.

   (xvi) _____ Every problem that can be mathematically defined has an algorithmic solution.

   (xvii) _____ The intersection of two undecidable languages is always undecidable.

   (xviii) _____ Every $\mathcal{NP}$ language is decidable.

(xix) ______ The intersection of two $\mathcal{NP}$ languages must be $\mathcal{NP}$.

(xx) Deleted.

(xxi) ______ $\mathcal{NC} = \mathcal{P}$.

(xxii) ______ $\mathcal{P} = \mathcal{NP}$.

(xxiii) ______ $\mathcal{NP} = \mathcal{P}$-SPACE

(xxiv) ______ $\mathcal{P}$-SPACE = EXP-TIME

(xxv) ______ EXP-TIME = EXP-SPACE

(xxvi) ______ NC = EXP-SPACE

(xxvii) ______ The traveling salesman problem (TSP) is $\mathcal{NP}$-complete.

(xxviii) ______ The knapsack problem is $\mathcal{NP}$-complete.

(xxix) ______ The language consisting of all satisfiable Boolean expressions is $\mathcal{NP}$-complete.

(xxx) ______ The Boolean Circuit Problem is in $\mathcal{P}$.

(xxxi) ______ The Boolean Circuit Problem is in $\mathcal{NC}$.

(xxxii) Deleted.

(xxxiii) ______ The language consisting of all strings over $\{a, b\}$ which have more a's than b's is context-free.

(xxxiv) ______ 2-SAT is $\mathcal{P}$-TIME.

(xxxv) ______ 3-SAT is $\mathcal{P}$-TIME.

(xxxvi) ______ Primality is $\mathcal{P}$-TIME.

(xxxvii) ______ There is a $\mathcal{P}$-TIME reduction of the halting problem to 3-SAT.

(xxxviii) ______ There is a $\mathcal{P}$-TIME reduction of the partition problem to 3-SAT.

(xxxix) ______ Every context-free language is in $\mathcal{P}$.

(xl) ______ Every context-free language is in $\mathcal{NC}$.

(xli) ______ Addition of binary numerals is in $\mathcal{NC}$.

(xlii) ______ Every context-sensitive language is in $\mathcal{P}$.

(xliii) ______ Every language generated by a general grammar is recursive.

(xliv) ______ The problem of whether two given context-free grammars generate the same language is decidable.

(xlv) ______ The language of all fractions (using base 10 numeration) whose values are less than π is decidable. (A *fraction* is a string. "314/100" is in the language, but "22/7" is not.)

(xlvi) ______ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary ("caveman") numeral.

(xlvii) ______ For any two languages L_1 and L_2, if L_2 is undecidable and there is a recursive reduction of L_1 to L_2, then L_1 must be undecidable.

(xlviii) ______ If P is a mathematical proposition that can be written using string of length n, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

As you may have learned, there is a formal language which can be used to write any mathematical proposition as well as any proof of any mathematical proposition, and an algorithm exists that can check the correctness of such a proof. In 1978, Jack Milnor https://en.wikipedia.org/wiki/John_Milnor told me that in the future no proof will be accepted unless it can be verified by a computer.

As you may have learned, there is a formal language which can be used to write

(xlix) Deleted.

(l) ______ Every bounded function is recursive.

(li) ______ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(lii) ______ Recall that if $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$. Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

(liii) ______ Every language is enumerable.

(liv) ______ If a language L is undecidable, then there can be no machine that enumerates L.

(lv) ______ There is a non-recursive function which grows faster than any recursive function.

(lvi) ______ There exists a machine[1] that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(lvii) ______ There is a non-recursive function which grows There is a machine which outputs the correct answer to the $\mathcal{P} = \mathcal{NP}$ question.

(lviii) ______ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(lix) Deleted

(lx) ______ **Rush Hour**, the puzzle sold in game stores everywhere, generalized to a board of arbitrary size, is $\mathcal{NP}$

(lxi) ______ The regular expression equivalence problem is $\mathcal{NP}$.

(lxii) ______ The intersection of two context-free languages is always context-free.

[1] As always in automata theory, "machine" means abstract machine, a mathematical object whose memory and running time are **not** constrained by the size and lifetime of the known (or unknown) universe, or any other physical laws. If we want to discuss the kind of machine that exists (or could exist) physically, we call it a "physical machine."

(lxiii) ______ The intersection of two regular languages is always regular.

(lxiv) ______ The intersection of a context-free language and a regular language is always context-free.

(lxv) ______ $\{a^i b^j c^i d^j\}$ is context-free.

(lxvi) ______ $\{a^i b^i c^j d^j\}$ is context-free.

(lxvii) ______ $\{a^i b^j c^j d^i\}$ is context-free.

2. [10 points] Fill in the blank. If L_1 is undecidable and if R is a reduction of L_1 to L_2 and if R is ________________, then L_2 is undecidable.

3. [20 points] State the pumping lemma for context-free languages.

4. [20 points] Use the pumping lemma for context-free languages to prove that the language $L = \{a^n b^n c^n\}$ is not context-free.

5. [20 points] Prove that a language is recursively enumerable if and only if it is accepted by some machine.

6. [20 points] Prove that the halting problem is undecidable.

7. [20 points] Give a definition of each of these $\mathcal{NP}$-complete languages/problems.

   (a) SAT

   (b) 3-SAT

   (c) Independent Set

   (d) Subset Sum

   (e) Partition

8. [20 points] Give a general (unrestricted) grammar for the language consisting of all string of 1's of length a power of 2, that is, $\{1^{2^n}\}$

9. [20 points] Give one of these polynomial time reductions (your choice).

   (a) 3-SAT to Independent Set.

   (b) Independent Set to Subset Sum

   (c) Subset Sum to Partition

10. [20 points] Prove that the context-free grammar equivalence problem is co-RE.